Tutorial 4: Minterms and Maxterms

- In Boolean algebra, a function can be expressed in two main canonical forms:
  - Sum of Products (SOP): Logical OR of minterms where the function equals 1.
  - Product of Sums (POS): Logical AND of maxterms where the function equals 0.

- A minterm represents one combination producing logic 1, while a maxterm represents one combination producing logic 0.

Question 1: For the given truth table with three variables A, B, and C, and an example function F:
Given truth table for F(A, B, C):

A B C | F
0 0 0 | 1
0 0 1 | 0
0 1 0 | 1
0 1 1 | 0
1 0 0 | 1
1 0 1 | 0
1 1 0 | 1
1 1 1 | 0

Answer the following:
(a) Express the function F in Sum of Products (SOP) form.
(b) Express the function F in Product of Sums (POS) form.

Answers:

F = 1 for minterms m(0, 2, 4, 6).

a) SOP form: F(A,B,C) = Σm(0,2,4,6) = A'B'C' + A'BC' + AB'C' + ABC'

b) POS form: F(A,B,C) = ΠM(1,3,5,7) = (A+B+C')(A+B'+C')(A'+B+C')(A'+B'+C')

Question2: Convert the following Boolean functions from a canonical sum-of-products form to a standard simplified product-of-sums form and vice versa.

a) F(x,y,z) = Σ(0,2,4,5)

b) F(x,y,z) = Σ(1,2,3,7)

c) F(x,y,z) = Π(1,3,4)

Answers:

Convert between canonical forms:

a) $F(x,y,z) = \Sigma(0,2,4,5) \rightarrow F = \Pi M(1,3,6,7)$

b) $F(x,y,z) = \Sigma(1,2,3,7) \rightarrow F = \Pi M(0,4,5,6)$

c) $F(x,y,z) = \Pi(1,3,4) \rightarrow F = \Sigma m(0,2,5,6,7)$

Question3: Complete the following questions:

1. In Boolean algebra, a minterm is defined as ---------

Ans: A product term that produces a logic 1 for exactly one combination of inputs.
Or A minterm gives output 1 for one input combination.

2. A + B + C is a -------term in a Boolean expression with variables A, B, and C.

Ans: Maxterm.

3. A' · B · C' is a -------term in a Boolean expression with variables A, B, and C.

Ans: Minterm.

4. If a Boolean function F(A,B,C) is represented as the sum of minterms $\Sigma m(1, 3, 5)$, does it imply that F is ---- for input combinations 1, 3, and 5.

Ans: 1

5. If a Boolean function F(A,B,C) is represented as the sum of minterms $\Pi M(0,2,4)$, does it imply that F is ---- for input combinations 1, 3, 5, 6, and 7, and imply that F is ---- for input combinations 0, 2 and 4.

Ans: 1, 0

6. A · B and A' · B' are examples that represent --- terms for variables A and B?

Ans: Minterms.

7. -------is A sum term that produces a logic 0 for exactly one combination of inputs.

Ans: Maxterm.

8. Write a function that represents an example of a sum of minterms?

Ans: (A) F(A,B) = A · B + A’ · B

Or F(A,B) = Σm(, , ,)

9. The maxterm M2 for variables A and B is --------

Ans: Maxterm M2 = (A + B').

10. The expression F(A,B,C) =ΠM(0,2,4) imply that F is --- for input combinations -----.

Ans: F is 0 for input combinations 0, 2, and 4.

11. For a Boolean function with two variables A and B, which minterm corresponds to the combination A = 0, B = 1?

Ans: A=0,B=1 → A'·B.

Question 4: Simplify the following questions:

Q1. $F = x(\bar{x} + y) + x$

Ans:

$$F = x.\bar{x} + x.y + x$$
$$= 0 + x.y + x$$
$$= x(y + 1)$$
$$= x.1 = x$$

Using $x.\bar{x} = 0$

Apply the Absorption Law: $x.y + x = x$

Q2. $F = \overline{(x + y)}\,(\bar{x} + \bar{y})$

Ans

$F = (\bar{x}.\bar{y})(\bar{x} + \bar{y})$

$= \bar{x}.\bar{y}.\bar{x} + \bar{x}.\bar{y}.\bar{y}$

$= \bar{x}.\bar{y} + \bar{x}.\bar{y} = \bar{x}.\bar{y}$

Apply De Morgan's Law to the first term : $\overline{(x + y)} = \bar{x}.\bar{y}$

Distribute : $(\bar{x}.\bar{y})(\bar{x} + \bar{y})$

Simplify : $\bar{x}.\bar{y}.\bar{x} = \bar{x}.\bar{y}$, $\bar{x}.\bar{y}.\bar{y} = \bar{x}.\bar{y}$

Q3. $F = A.B.C + \bar{A} + A.\bar{B}.C$

Ans:

$F = A.C(B + \bar{B}) + \bar{A}$

$= A.C.(1) + \bar{A} = A.C + \bar{A}$

$= (\bar{A} + C)(\bar{A} + A)$

$= (\bar{A} + C)(1) = (\bar{A} + C)$

Q4. $F = A.\bar{B} + \overline{(\bar{A} + \bar{B} + C\bar{C})}$

Ans:

$F = A.\bar{B} + \overline{(\bar{A} + \bar{B} + 0)}$

$F = A.\bar{B} + \bar{\bar{A}}\,.\bar{\bar{B}}$

$= A.\bar{B} + A.B$

$= A(\bar{B} + B)$

$= A(1) = A$