



جامعة برج العرب التكنولوجية
BORG AL ARAB TECHNOLOGICAL UNIVERSITY



Digital Engineering

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Lecture 6

Circuit Minimization

- How do you think could we simplify these equations ??
- One possible solution is to use Boolean Algebra.
- Consider the sum of products equation:

$$f = A'BC' + A'BC + AB'C + ABC$$

- Search for any two similar terms except one complement ??

$$f = A'BC' + A'BC + AB'C + ABC$$

$$f = A'BC' + A'BC + AB'C + ABC$$

$$f = A'BC' + A'BC + AB'C + ABC$$

Circuit Minimization

- For each group apply the rule $XY + XY' = X(Y + Y') = X$.

$$f = A'BC' + A'BC + AB'C + ABC \rightarrow A'B(C' + C) = A'B$$

$$f = A'BC' + A'BC + AB'C + ABC \rightarrow (A' + A)BC = BC$$

$$f = A'BC' + A'BC + AB'C + ABC \rightarrow A(B' + B)C = AC$$

- If you found two intersecting group, duplicate the common term, applying the rule $X + X = X$:

$$f = A'BC' + A'BC + A'BC + AB'C + ABC + ABC$$

- Group the similar terms:

$$f = (A'BC' + A'BC) + (A'BC + ABC) + (AB'C + ABC)$$

Circuit Minimization

- Group the similar terms:

$$f = (A'BC' + A'BC) + (A'BC + ABC) + (AB'C + ABC)$$

- Apply the rule $XY + XY' = X(Y + Y') = X$, then:

$$f = A'B(C' + C) + (A' + A)BC + A(B' + B)C$$

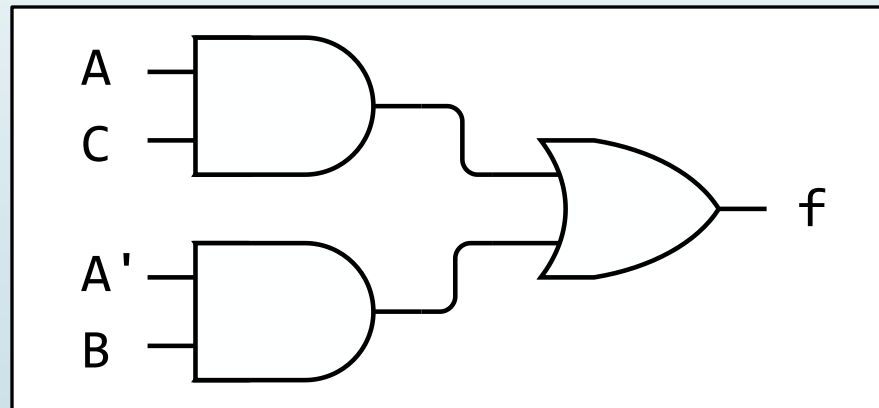
- Then: $f = A'B + BC + AC$

- Remember the redundancy rule: $XY + X'Z + YZ = XY + X'Z$

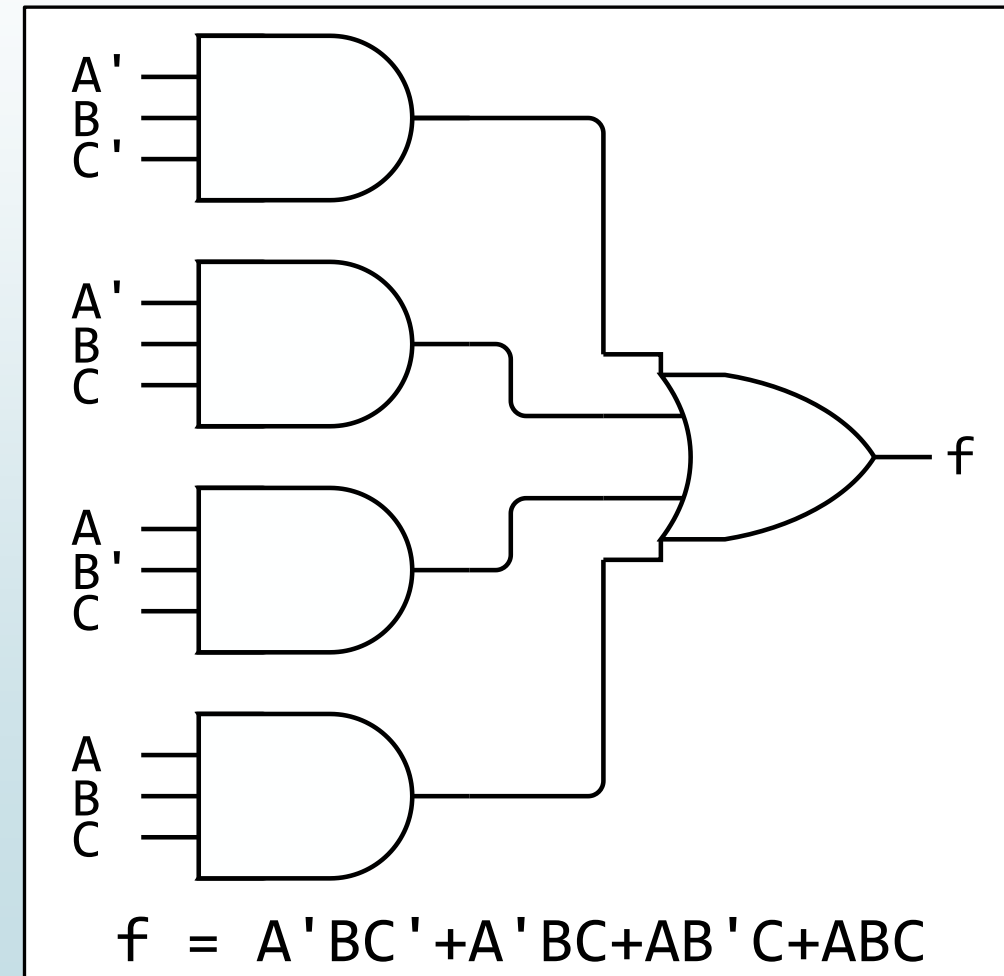
- Then: $f = AC + A'B + BC = AC + A'B$

Circuit Minimization

- The final simplified function:
- Then: $f = AC + A'B$
- The circuit implementation:

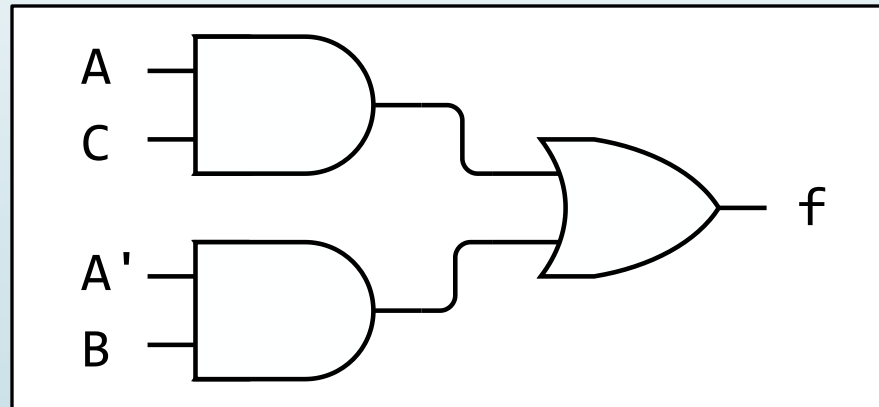


- Compare to the original one.



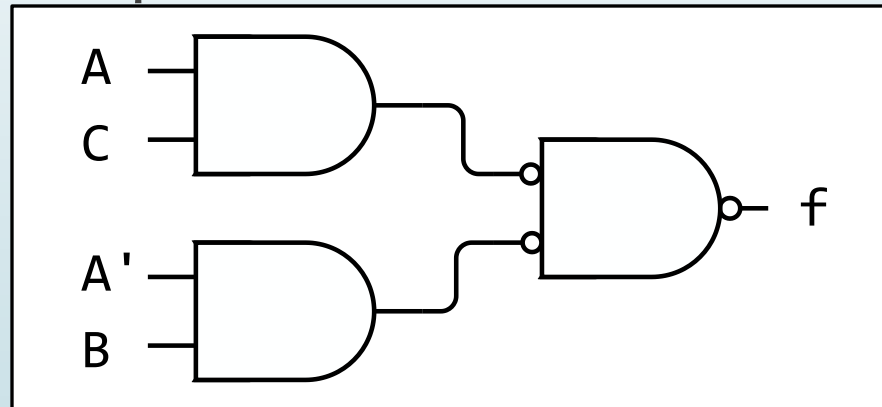
Boolean Algebra

- To further reduce the cost: use graphical DeMorgans law on the OR gate:



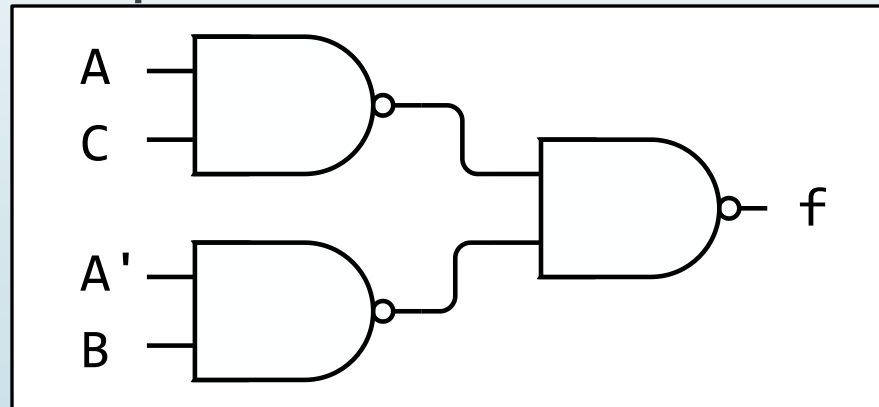
Boolean Algebra

- To further reduce the cost: use graphical DeMorgans law on the OR gate:
 - Change the OR to AND and complement all its inputs and output.



Boolean Algebra

- To further reduce the cost: use graphical DeMorgans law on the OR gate:
 - Change the OR to AND and complement all its inputs and output.



- This is a NAND-NAND circuit that is cheaper than the AND-OR circuit, smaller and consumes less power.

Karnaugh Map

Boolean Algebra Minimization

- Recall the prime number detector designed before:

$$f = \sum_{A,B,C} m(2,3,5,7)$$

- Resolve the minterms:

$$f = A'BC' + A'BC + AB'C + ABC$$

$$f = A'B + BC + AC$$

- Search for any two similar terms except one complement.
- Apply the rule $XY + XY' = X(Y + Y') = X$

Karnaugh Map

- The previous example contains only 3 input variables.
- The simplification will be harder and longer with increasing the number of input variables.
- The American scientist Maurice Karnaugh invented his map for doing this grouping visually for faster simplification.

Karnaugh Map

- Assume 3-input variables. Consider the 8 minterms list.
- Assume a table of 8 cells.

A	B	C	Minterms
0	0	0	$A'B'C' = m_0$
0	0	1	$A'B'C = m_1$
0	1	0	$A'BC' = m_2$
0	1	1	$A'BC = m_3$
1	0	0	$AB'C' = m_4$
1	0	1	$AB'C = m_5$
1	1	0	$ABC' = m_6$
1	1	1	$ABC = m_7$

Karnaugh Map

- Assume that we will put m_0 in the first cell. Find the similar minterms except one complement ??

$\emptyset$ $A'B'C'$	

A	B	C	Minterms
$\emptyset$	$\emptyset$	$\emptyset$	$A'B'C' = m_0$
$\emptyset$	$\emptyset$	1	$A'B'C = m_1$
$\emptyset$	1	$\emptyset$	$A'BC' = m_2$
$\emptyset$	1	1	$A'BC = m_3$
1	$\emptyset$	$\emptyset$	$AB'C' = m_4$
1	$\emptyset$	1	$AB'C = m_5$
1	1	$\emptyset$	$ABC' = m_6$
1	1	1	$ABC = m_7$

Karnaugh Map

- You will find m_1 . Put it beside m_0 . Search for other similar minterm.

0 $A'B'C'$	
1 $A'B'C$	

A	B	C	Minterms
0	0	0	$A'B'C' = m_0$
0	0	1	$A'B'C = m_1$
0	1	0	$A'BC' = m_2$
0	1	1	$A'BC = m_3$
1	0	0	$AB'C' = m_4$
1	0	1	$AB'C = m_5$
1	1	0	$ABC' = m_6$
1	1	1	$ABC = m_7$

Karnaugh Map

- You will find m_2 . Put it beside m_0 . Search for other similar minterm.

0 $A'B'C'$	
1 $A'B'C$	
2 $A'BC'$	

A	B	C	Minterms
0	0	0	$A'B'C' = m_0$
0	0	1	$A'B'C = m_1$
0	1	0	$A'BC' = m_2$
0	1	1	$A'BC = m_3$
1	0	0	$AB'C' = m_4$
1	0	1	$AB'C = m_5$
1	1	0	$ABC' = m_6$
1	1	1	$ABC = m_7$

Karnaugh Map

- You will find m_4 . Put it beside m_0 . No more minterms similar to m_0 .

0 $A'B'C'$	4 $AB'C'$
1 $A'B'C$	
2 $A'BC'$	

A	B	C	Minterms
0	0	0	$A'B'C' = m_0$
0	0	1	$A'B'C = m_1$
0	1	0	$A'BC' = m_2$
0	1	1	$A'BC = m_3$
1	0	0	$AB'C' = m_4$
1	0	1	$AB'C = m_5$
1	1	0	$ABC' = m_6$
1	1	1	$ABC = m_7$

Karnaugh Map

- Similarly, fill in all the table so that each two adjacent cells are different in only one complement.
- Check them ...

⁰ $A'B'C'$	⁴ $AB'C'$
¹ $A'B'C$	⁵ $AB'C$
³ $A'BC$	⁷ ABC
² $A'BC'$	⁶ ABC'

A	B	C	Minterms
0	0	0	$A'B'C' = m_0$
0	0	1	$A'B'C = m_1$
0	1	0	$A'BC' = m_2$
0	1	1	$A'BC = m_3$
1	0	0	$AB'C' = m_4$
1	0	1	$AB'C = m_5$
1	1	0	$ABC' = m_6$
1	1	1	$ABC = m_7$

Karnaugh Map

► Note the common letter in each row and column. Check it ...

	A'	A	
B'	⁰ A'B'C'	⁴ AB'C'	C'
B'	¹ A'B'C	⁵ AB'C	C
B	³ A'BC	⁷ ABC	C
B	² A'BC'	⁶ ABC'	C'

A	B	C	Minterms
0	0	0	A'B'C' = m ₀
0	0	1	A'B'C = m ₁
0	1	0	A'BC' = m ₂
0	1	1	A'BC = m ₃
1	0	0	AB'C' = m ₄
1	0	1	AB'C = m ₅
1	1	0	ABC' = m ₆
1	1	1	ABC = m ₇

Karnaugh Map

► Note the common letter in each row and column. Check it ...

	A'	A	
B'	⁰ A'B'C'	⁴ AB'C'	C'
	¹ A'B'C	⁵ AB'C	
B	³ A'BC	⁷ ABC	C
	² A'BC'	⁶ ABC'	

A	B	C	Minterms
0	0	0	$A'B'C' = m_0$
0	0	1	$A'B'C = m_1$
0	1	0	$A'BC' = m_2$
0	1	1	$A'BC = m_3$
1	0	0	$AB'C' = m_4$
1	0	1	$AB'C = m_5$
1	1	0	$ABC' = m_6$
1	1	1	$ABC = m_7$

Karnaugh Map

- The adjacent cells are different in only one complement. So, we can group any two adjacent cells.

B'	A'	A	C'
	$\emptyset$ $A'B'C'$	4 $AB'C'$	
B	1 $A'BC$	5 $AB'C$	C
	3 $A'BC$	7 ABC	
B	2 $A'BC'$	6 ABC'	C'

A	B	C	Minterms
$\emptyset$	$\emptyset$	$\emptyset$	$A'B'C' = m_{\emptyset}$
$\emptyset$	$\emptyset$	1	$A'B'C = m_1$
$\emptyset$	1	$\emptyset$	$A'BC' = m_2$
$\emptyset$	1	1	$A'BC = m_3$
1	$\emptyset$	$\emptyset$	$AB'C' = m_4$
1	$\emptyset$	1	$AB'C = m_5$
1	1	$\emptyset$	$ABC' = m_6$
1	1	1	$ABC = m_7$

Karnaugh Map

► Example: $A'B'C + AB'C = B'C$

► The result is the common letters in the group.

B'	A'	A	C'
	$\begin{matrix} 0 \\ A'B'C' \end{matrix}$	$\begin{matrix} 4 \\ AB'C' \end{matrix}$	
B	$\begin{matrix} 1 \\ A'B'C \end{matrix}$	$\begin{matrix} 5 \\ AB'C \end{matrix}$	C
	$\begin{matrix} 3 \\ A'BC \end{matrix}$	$\begin{matrix} 7 \\ ABC \end{matrix}$	
	$\begin{matrix} 2 \\ A'BC' \end{matrix}$	$\begin{matrix} 6 \\ ABC' \end{matrix}$	C'

A	B	C	Minterms
0	0	0	$A'B'C' = m_0$
0	0	1	$A'B'C = m_1$
0	1	0	$A'BC' = m_2$
0	1	1	$A'BC = m_3$
1	0	0	$AB'C' = m_4$
1	0	1	$AB'C = m_5$
1	1	0	$ABC' = m_6$
1	1	1	$ABC = m_7$

Karnaugh Map

► We can also group 4 cells. Example:

$$(A'B'C' + AB'C') + (A'B'C + AB'C) = B'C + B'C' = B'$$

	A'	A	
B'	⁰ A'B'C'	⁴ AB'C'	C'
	¹ A'B'C	⁵ AB'C	C
B	³ A'BC	⁷ ABC	
	² A'BC'	⁶ ABC'	C'

A	B	C	Minterms
0	0	0	A'B'C' = m ₀
0	0	1	A'B'C = m ₁
0	1	0	A'BC' = m ₂
0	1	1	A'BC = m ₃
1	0	0	AB'C' = m ₄
1	0	1	AB'C = m ₅
1	1	0	ABC' = m ₆
1	1	1	ABC = m ₇

Karnaugh Map

- This table is the Karnaugh Map (K-map).
- We distributed the input side of truth table.

		A'		A			
		0		4			
B'		A'B'C'		AB'C'		C'	
	1		A'B'C		AB'C		C
B		3		7			
		A'BC		ABC			
	2		A'BC'		ABC'		C'

A	B	C	Minterms
0	0	0	$A'B'C' = m_0$
0	0	1	$A'B'C = m_1$
0	1	0	$A'BC' = m_2$
0	1	1	$A'BC = m_3$
1	0	0	$AB'C' = m_4$
1	0	1	$AB'C = m_5$
1	1	0	$ABC' = m_6$
1	1	1	$ABC = m_7$

Karnaugh Map

- Replace each input with its corresponding output.
- Note the order of ones and zeros.

	A'	A	
B'	0 0	4 0	C'
	1 0	5 1	
B	3 1	7 1	C
	2 1	6 0	

A	B	C	Minterms	f
0	0	0	$A'B'C' = m_0$	0
0	0	1	$A'B'C = m_1$	0
0	1	0	$A'BC' = m_2$	1
0	1	1	$A'BC = m_3$	1
1	0	0	$AB'C' = m_4$	0
1	0	1	$AB'C = m_5$	1
1	1	0	$ABC' = m_6$	0
1	1	1	$ABC = m_7$	1

Karnaugh Map

- Previously, we considered only the active minterms.
- Now, we will consider only the ones inside the map.

		A'	A	
B'	0	0	0	c'
	1	0	1	c
B	3	1	1	
	2	1	0	c'

A	B	C	f
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Karnaugh Map

► Give the letters of this group:

► $f = ??$

	A'	A	
B'	0 0	4 0	C'
	1 0	5 1	C
B	3 1	7 1	
	2 1	6 0	C'

A	B	C	f
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Karnaugh Map

► Give the letters of this group:

► $f = AC + ??$

	A'	A	
B'	0 0	4 0	C'
	1 0	5 1	C
B	3 1	7 1	
	2 1	6 0	C'

A	B	C	f
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Karnaugh Map

► This is the only left group, do not consider it. We have all 1s.

► $f = AC + A'B$

	A'	A	
B'	0 0	4 0	C'
	1 0	5 1	C
B	3 1	7 1	
	2 1	6 0	C'

A	B	C	f
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Karnaugh Map

► This is the final simplified equation. Compare with before.

► $f = AC + A'B$

		A'	A	
B'	0	0	0	
	1	0	1	C
B	3	1	1	
	2	1	0	C'

A	B	C	f
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Steps Summary

- Given a truth table, it is required to find its equation.
- Draw the Karnaugh Map for 3-inputs

A	B	C	f
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

	A'	A	
B'	0	4	C'
	1	5	
B	3	7	C
	2	6	C'

Steps Summary

- Given a truth table, it is required to find its equation.
- Draw the Karnaugh Map for 3-inputs
- Put the function output inside the cells.
- Find the minimum number of the largest groups that cover all the ones.
- Groups must be 2^n elements, i.e.: 1,2,4,8
- The equations is the summation of these groups letters.
- $f = AC + A'B$

A	B	C	f
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

	A'	A	
	0	4	C'
B'	0	0	
	1	5	
	0	1	C
	3	7	
	1	1	
B	2	6	C'
	0	0	

Grouping Notes

- Consider this case. You have 2 options for grouping.
- Select the minimum number of the largest groups.
- This leads to the result without extra simplification.

$$f = AB' + AB = A$$

	A'	A	
B'	0 0	4 1	C'
	1 0	5 1	C
B	3 0	7 1	C
	2 0	6 1	C'



$$f = A$$

	A'	A	
B'	0 0	4 1	C'
	1 0	5 1	C
B	3 0	7 1	C
	2 0	6 1	C'



Grouping Notes

- Consider this case. You have 2 options for grouping.
- Select the minimum number of the largest groups.
- This leads to the result without extra simplification.
- $f = ? + ?$



	A'	A	
B'	0 0	4 1	C'
	1 1	5 1	
B	3 1	7 1	C
	2 0	6 1	C'



	A'	A	
B'	0 0	4 1	C'
	1 1	5 1	
B	3 1	7 1	C
	2 0	6 1	C'

Grouping Notes

- Consider this case. You have 2 options for grouping.
- Select the minimum number of the largest groups.
- This leads to the result without extra simplification.
- $f = A + A' C$

	A'	A	
B'	0 0	4 1	C'
	1 1	5 1	C
B	3 1	7 1	C
	2 0	6 1	C'



$f = ? + ?$

	A'	A	
B'	0 0	4 1	C'
	1 1	5 1	C
B	3 1	7 1	C
	2 0	6 1	C'



Grouping Notes

- Consider this case. You have 2 options for grouping.
- Select the minimum number of the largest groups.
- This leads to the result without extra simplification.
- $f = A + A' C$

	A'	A	
B'	0 0	4 1	C'
	1 1	5 1	C
B	3 1	7 1	C
	2 0	6 1	C'



	A'	A	
B'	0 0	4 1	C'
	1 1	5 1	C
B	3 1	7 1	C
	2 0	6 1	C'



Grouping Notes

- Consider this case. You have 2 options for grouping.
- Select the minimum number of the largest groups.
- This leads to the result without extra simplification.
- $f = A + A' C = (A + A') (A + C) = A + C$

	A'	A	
B'	0 0	4 1	C'
	1 1	5 1	C
B	3 1	7 1	C
	2 0	6 1	C'



	A'	A	
B'	0 0	4 1	C'
	1 1	5 1	C
B	3 1	7 1	C
	2 0	6 1	C'



Grouping Notes

- Consider this case. You have 2 options for grouping.
- Select the minimum number of the largest groups.
- This leads to the result without extra simplification.
- $f = A + A' C = (A + A') (A + C) = A + C$

	A'	A	
B'	0 0	4 1	C'
	1 1	5 1	
B	3 1	7 1	C
	2 0	6 1	C'

	A'	A	
B'	0 0	4 1	C'
	1 1	5 1	
B	3 1	7 1	C
	2 0	6 1	C'

Two Inputs

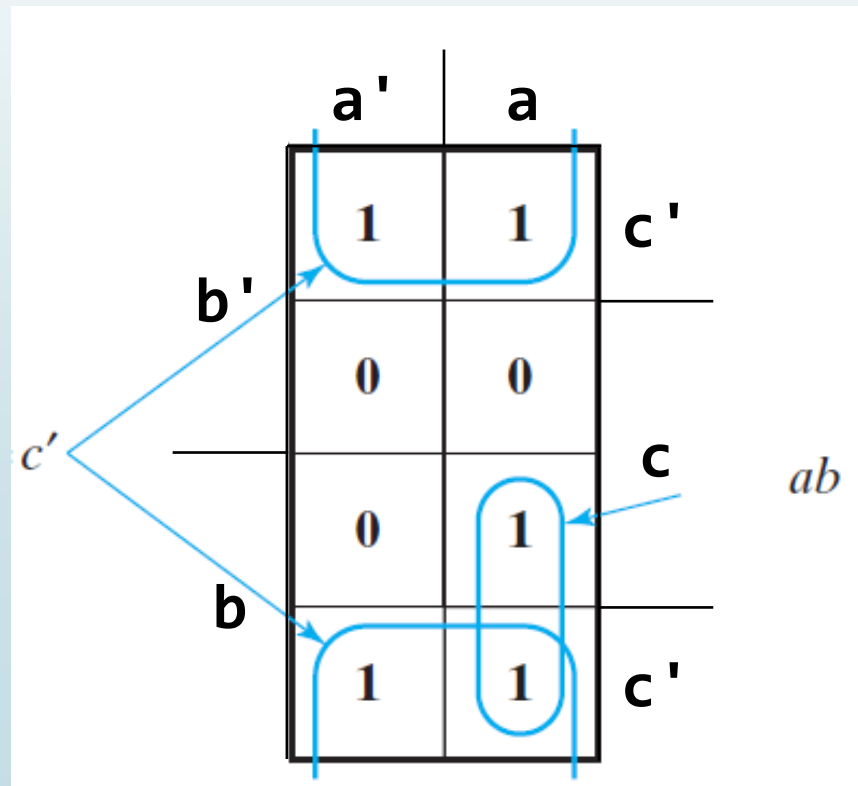
- In case of 2- input functions. It might be easier to resolve it without Karnaugh map.
- Consider the given truth table.
- Using minterms only: $f = A'B' + A'B = A'$
- However, you could use the Karnaugh map in this way:
 - Draw the 2-inputs K-map like this:
 - $f = A'$

A	B	f
0	0	1
0	1	1
1	0	0
1	1	0

	A'	A
B'	0 1	2 0
B	1 1	3 0

Examples from Textbook

$$\blacksquare f = ab + c'$$



Examples from Textbook

■ The clear cells are zeros by default.

■ $f = xy + x'z + yz = xy + x'z$

$f = x'z + xy$

