



جامعة برج العرب التكنولوجية
BORG AL ARAB TECHNOLOGICAL UNIVERSITY



Digital Engineering

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Lecture 4

Minterms and Maxterms

Minterms and Maxterms

- Remember, we are now designing a circuit. It is required to find its circuit diagram.
- Previously, we got the truth table which is the first step in designing a logic circuit.
- The next step is to convert this truth table into an equation, so that we could implement it using the basic gates.
- This is done using minterms and maxterms.
- The minterms and maxterms are ways for expressing an input case as Boolean function.

Minterms and Maxterms

- Given a truth table. It is required to find the output equation.
- Example: The prime number detector circuit.

A	B	C	f
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Minterms and Maxterms

► Ignore the output side.

A	B	C
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

Minterms and Maxterms

► How to make an input results in 1 on AND gate?

A	B	C
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

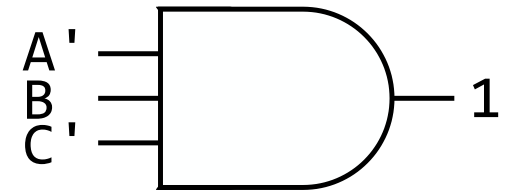
Minterms and Maxterms

► How to make an input results in 1 on AND gate?

► Example: When $A=0$, $B=1$, $C=0$, we need the AND output =1

A	B	C
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

$A'BC'$



Minterms and Maxterms

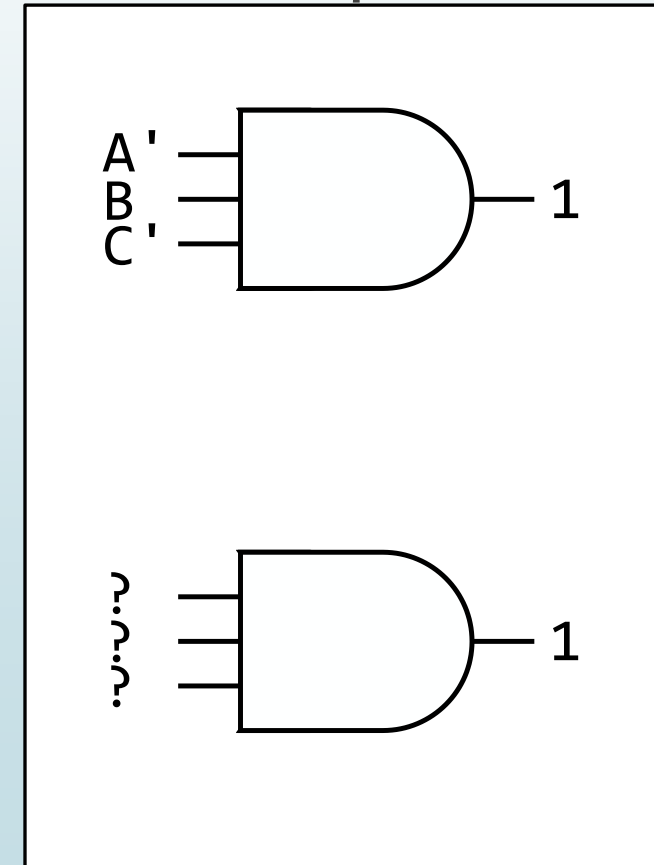
► How to make an input results in 1 on AND gate?

► Example: When $A=1$, $B=0$, $C=1$, we need the AND output =1

A	B	C
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

$A'BC'$

??



Minterms and Maxterms

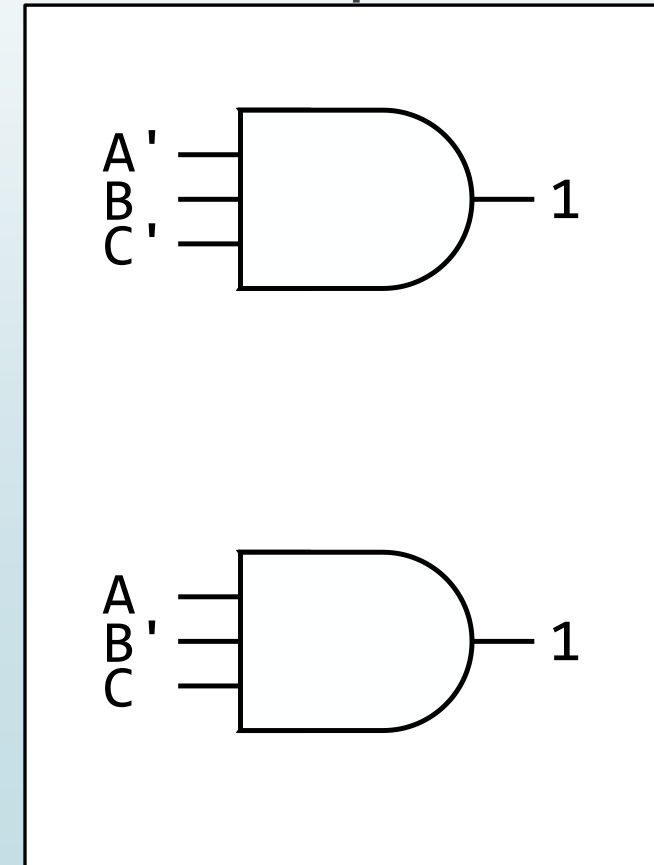
► How to make an input results in 1 on AND gate?

► Example: When $A=1$, $B=0$, $C=1$, we need the AND output =1

A	B	C
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

$A'BC'$

$AB'C$



Minterms and Maxterms

- How to make an input results in 1 on AND gate?
- Repeat for all the input combinations.

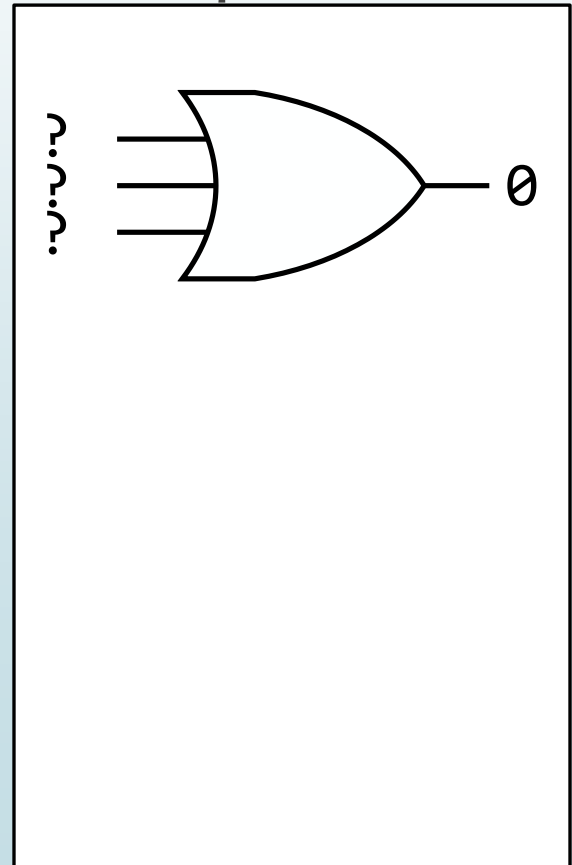
A	B	C	
0	0	0	$A'B'C'$
0	0	1	$A'B'C$
0	1	0	$A'BC'$
0	1	1	$A'BC$
1	0	0	$AB'C'$
1	0	1	$AB'C$
1	1	0	ABC'
1	1	1	ABC

Minterms and Maxterms

► How to make an input results in 0 on OR gate?

► Example: When $A=0$, $B=0$, $C=1$, we need the OR output =1

A	B	C	
0	0	0	$A'B'C'$
0	0	1	$A'B'C$
0	1	0	$A'BC'$
0	1	1	$A'BC$
1	0	0	$AB'C'$
1	0	1	$AB'C$
1	1	0	ABC'
1	1	1	ABC

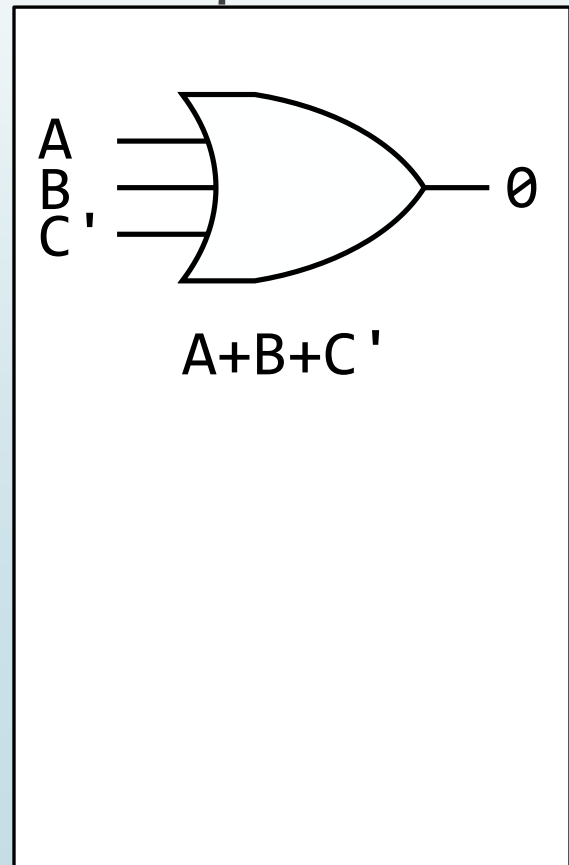


Minterms and Maxterms

► How to make an input results in 0 on OR gate?

► Example: When $A=0$, $B=0$, $C=1$, we need the OR output = 1

A	B	C	
0	0	0	$A'B'C'$
0	0	1	$A'B'C$
0	1	0	$A'BC'$
0	1	1	$A'BC$
1	0	0	$AB'C'$
1	0	1	$AB'C$
1	1	0	ABC'
1	1	1	ABC

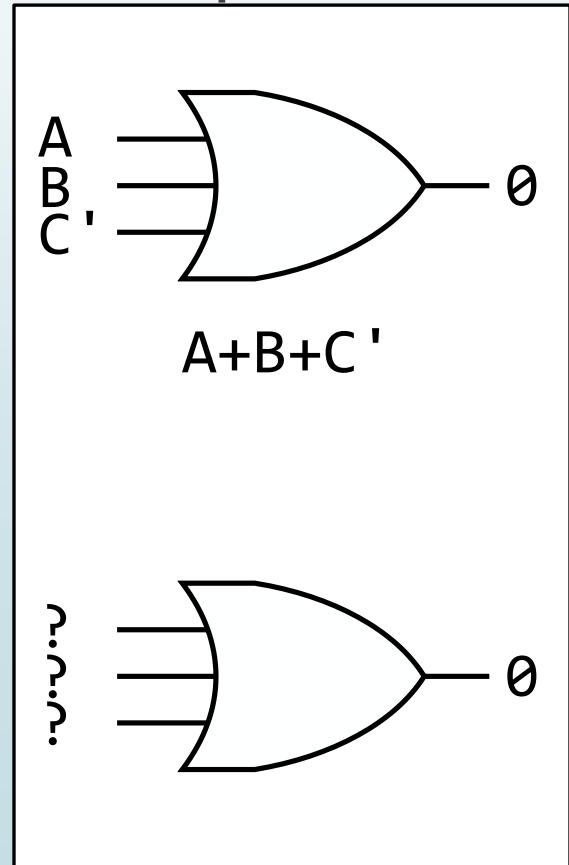


Minterms and Maxterms

► How to make an input results in 0 on OR gate?

► Example: When $A=1$, $B=1$, $C=0$, we need the OR output = 1

A	B	C	
0	0	0	$A'B'C'$
0	0	1	$A'B'C$
0	1	0	$A'BC'$
0	1	1	$A'BC$
1	0	0	$AB'C'$
1	0	1	$AB'C$
1	1	0	ABC'
1	1	1	ABC

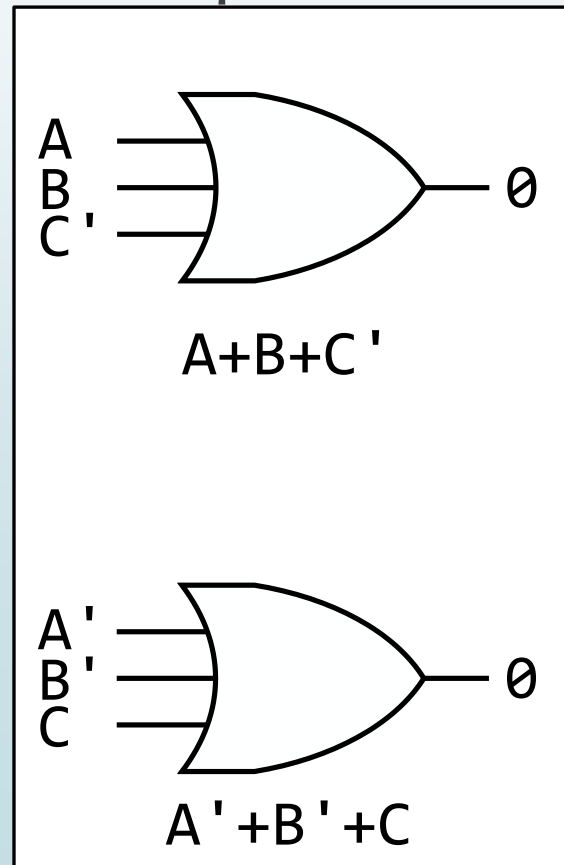


Minterms and Maxterms

► How to make an input results in 0 on OR gate?

► Example: When $A=1$, $B=1$, $C=0$, we need the OR output =1

A	B	C	
0	0	0	$A'B'C'$
0	0	1	$A'B'C$
0	1	0	$A'BC'$
0	1	1	$A'BC$
1	0	0	$AB'C'$
1	0	1	$AB'C$
1	1	0	ABC'
1	1	1	ABC



Minterms and Maxterms

- How to make an input results in 0 on OR gate?
- Repeat for all the input combinations.

A	B	C		
0	0	0	$A'B'C'$	$A+B+C$
0	0	1	$A'B'C$	$A+B+C'$
0	1	0	$A'BC'$	$A+B'+C$
0	1	1	$A'BC$	$A+B'+C'$
1	0	0	$AB'C'$	$A'+B+C$
1	0	1	$AB'C$	$A'+B+C'$
1	1	0	ABC'	$A'+B'+C$
1	1	1	ABC	$A'+B'+C'$

Minterms and Maxterms

► Names of terms:

A	B	C		
0	0	0	$A'B'C'$	$A+B+C$
0	0	1	$A'B'C$	$A+B+C'$
0	1	0	$A'BC'$	$A+B'+C$
0	1	1	$A'BC$	$A+B'+C'$
1	0	0	$AB'C'$	$A'+B+C$
1	0	1	$AB'C$	$A'+B+C'$
1	1	0	ABC'	$A'+B'+C$
1	1	1	ABC	$A'+B'+C'$

Minterms and Maxterms

- The product terms are called Minterms.
- The sum terms are called Maxterms.

A	B	C	Minterms	Maxterms
0	0	0	$A'B'C'$	$A+B+C$
0	0	1	$A'B'C$	$A+B+C'$
0	1	0	$A'BC'$	$A+B'+C$
0	1	1	$A'BC$	$A+B'+C'$
1	0	0	$AB'C'$	$A'+B+C$
1	0	1	$AB'C$	$A'+B+C'$
1	1	0	ABC'	$A'+B'+C$
1	1	1	ABC	$A'+B'+C'$

Minterms and Maxterms

- The product terms are called Minterms (m).
- The sum terms are called Maxterms (M).

A	B	C	Minterms	Maxterms
0	0	0	$A'B'C' = m_0$	$A+B+C = M_0$
0	0	1	$A'B'C = m_1$	$A+B+C' = M_1$
0	1	0	$A'BC' = m_2$	$A+B'+C = M_2$
0	1	1	$A'BC = m_3$	$A+B'+C' = M_3$
1	0	0	$AB'C' = m_4$	$A'+B+C = M_4$
1	0	1	$AB'C = m_5$	$A'+B+C' = M_5$
1	1	0	$ABC' = m_6$	$A'+B'+C = M_6$
1	1	1	$ABC = m_7$	$A'+B'+C' = M_7$

Minterms

- Till now we just give an expression to describe the input.
- Now, it is required to find the equation of the circuit output.

A	B	C	Minterms	Maxterms
0	0	0	$A'B'C' = m_0$	$A+B+C = M_0$
0	0	1	$A'B'C = m_1$	$A+B+C' = M_1$
0	1	0	$A'BC' = m_2$	$A+B'+C = M_2$
0	1	1	$A'BC = m_3$	$A+B'+C' = M_3$
1	0	0	$AB'C' = m_4$	$A'+B+C = M_4$
1	0	1	$AB'C = m_5$	$A'+B+C' = M_5$
1	1	0	$ABC' = m_6$	$A'+B'+C = M_6$
1	1	1	$ABC = m_7$	$A'+B'+C' = M_7$

Minterms

- Now, consider the complete truth table.
- Example: The prime number detector circuit.

A	B	C	Minterms	Maxterms	f
0	0	0	$A'B'C' = m_0$	$A+B+C = M_0$	0
0	0	1	$A'B'C = m_1$	$A+B+C' = M_1$	0
0	1	0	$A'BC' = m_2$	$A+B'+C = M_2$	1
0	1	1	$A'BC = m_3$	$A+B'+C' = M_3$	1
1	0	0	$AB'C' = m_4$	$A'+B+C = M_4$	0
1	0	1	$AB'C = m_5$	$A'+B+C' = M_5$	1
1	1	0	$ABC' = m_6$	$A'+B'+C = M_6$	0
1	1	1	$ABC = m_7$	$A'+B'+C' = M_7$	1

Minterms

- Look at all the input cases that results in 1. The output f will be 1 in case of *the 2nd input OR the 3rd OR the 5th OR 7th*.

A	B	C	Minterms	Maxterms	f
0	0	0	$A'B'C' = m_0$	$A+B+C = M_0$	0
0	0	1	$A'B'C = m_1$	$A+B+C' = M_1$	0
0	1	0	$A'BC' = m_2$	$A+B'+C = M_2$	1
0	1	1	$A'BC = m_3$	$A+B'+C' = M_3$	1
1	0	0	$AB'C' = m_4$	$A'+B+C = M_4$	0
1	0	1	$AB'C = m_5$	$A'+B+C' = M_5$	1
1	1	0	$ABC' = m_6$	$A'+B'+C = M_6$	0
1	1	1	$ABC = m_7$	$A'+B'+C' = M_7$	1

Minterms

► Convert this sentence into equation:

$$f = (A'BC') + (A'BC) + (AB'C) + (ABC)$$

A	B	C	Minterms	Maxterms	f
0	0	0	$A'B'C' = m_0$	$A+B+C = M_0$	0
0	0	1	$A'B'C = m_1$	$A+B+C' = M_1$	0
0	1	0	$A'BC' = m_2$	$A+B'+C = M_2$	1
0	1	1	$A'BC = m_3$	$A+B'+C' = M_3$	1
1	0	0	$AB'C' = m_4$	$A'+B+C = M_4$	0
1	0	1	$AB'C = m_5$	$A'+B+C' = M_5$	1
1	1	0	$ABC' = m_6$	$A'+B'+C = M_6$	0
1	1	1	$ABC = m_7$	$A'+B'+C' = M_7$	1

Minterms

- Convert this sentence into equation:

$$f = (A'BC') + (A'BC) + (AB'C) + (ABC)$$

- Note that we used the minterm because it result in 1, while the maxterm result in 0.
- Then, the output f will be 1 in case of *the 2nd input OR the 3rd OR the 5th OR 7th*.
- This is the required expression.
- This is called **Sum of Products** form.
- Each of the 4 minterms is called **product term**. It is the logical product (AND) of many variables.

Sum of Products

- Try all the inputs (prime numbers) and check the output.

$$f = (A'BC') + (A'BC) + (AB'C) + (ABC)$$

A	B	C	Minterms	Maxterms	f
0	0	0	$A'B'C' = m_0$	$A+B+C = M_0$	0
0	0	1	$A'B'C = m_1$	$A+B+C' = M_1$	0
0	1	0	$A'BC' = m_2$	$A+B'+C = M_2$	1
0	1	1	$A'BC = m_3$	$A+B'+C' = M_3$	1
1	0	0	$AB'C' = m_4$	$A'+B+C = M_4$	0
1	0	1	$AB'C = m_5$	$A'+B+C' = M_5$	1
1	1	0	$ABC' = m_6$	$A'+B'+C = M_6$	0
1	1	1	$ABC = m_7$	$A'+B'+C' = M_7$	1

Sum of Products

- Try all the inputs (prime numbers) and check the output.

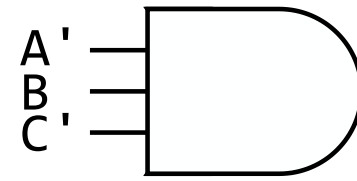
$$f = (A'BC') + (A'BC) + (AB'C) + (ABC)$$

A	B	C	Minterms	Maxterms	f
0	0	0	$A'B'C' = m_0$	$A+B+C = M_0$	0
0	0	1	$A'B'C = m_1$	$A+B+C' = M_1$	0
0	1	0	$A'BC' = m_2$	$A+B'+C = M_2$	1
0	1	1	$A'BC = m_3$	$A+B'+C' = M_3$	1
1	0	0	$AB'C' = m_4$	$A'+B+C = M_4$	0
1	0	1	$AB'C = m_5$	$A'+B+C' = M_5$	1
1	1	0	$ABC' = m_6$	$A'+B'+C = M_6$	0
1	1	1	$ABC = m_7$	$A'+B'+C' = M_7$	1

Logic Circuit

- Convert this equation into circuit:

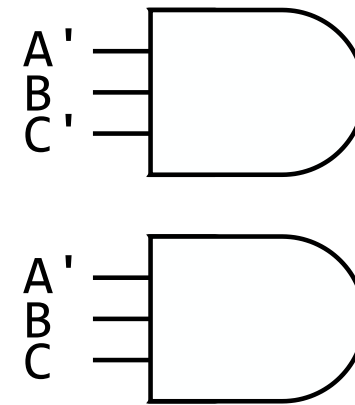
$$f = A'BC' + A'BC + AB'C + ABC$$



Logic Circuit

- Convert this equation into circuit:

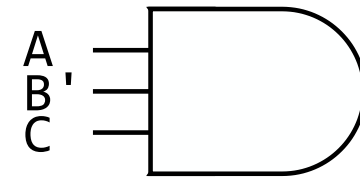
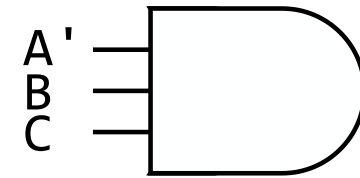
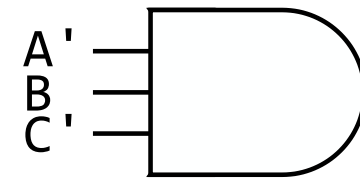
$$f = A'BC' + A'BC + AB'C + ABC$$



Logic Circuit

- Convert this equation into circuit:

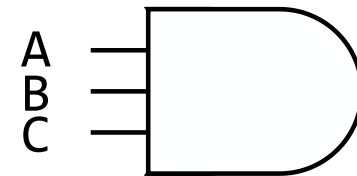
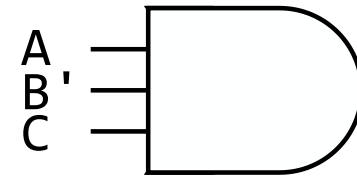
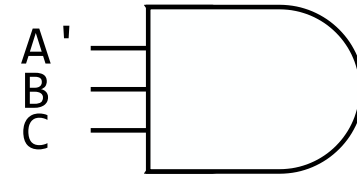
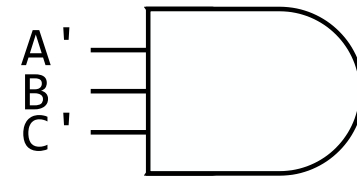
$$f = A'BC' + A'BC + AB'C + ABC$$



Logic Circuit

- Convert this equation into circuit:

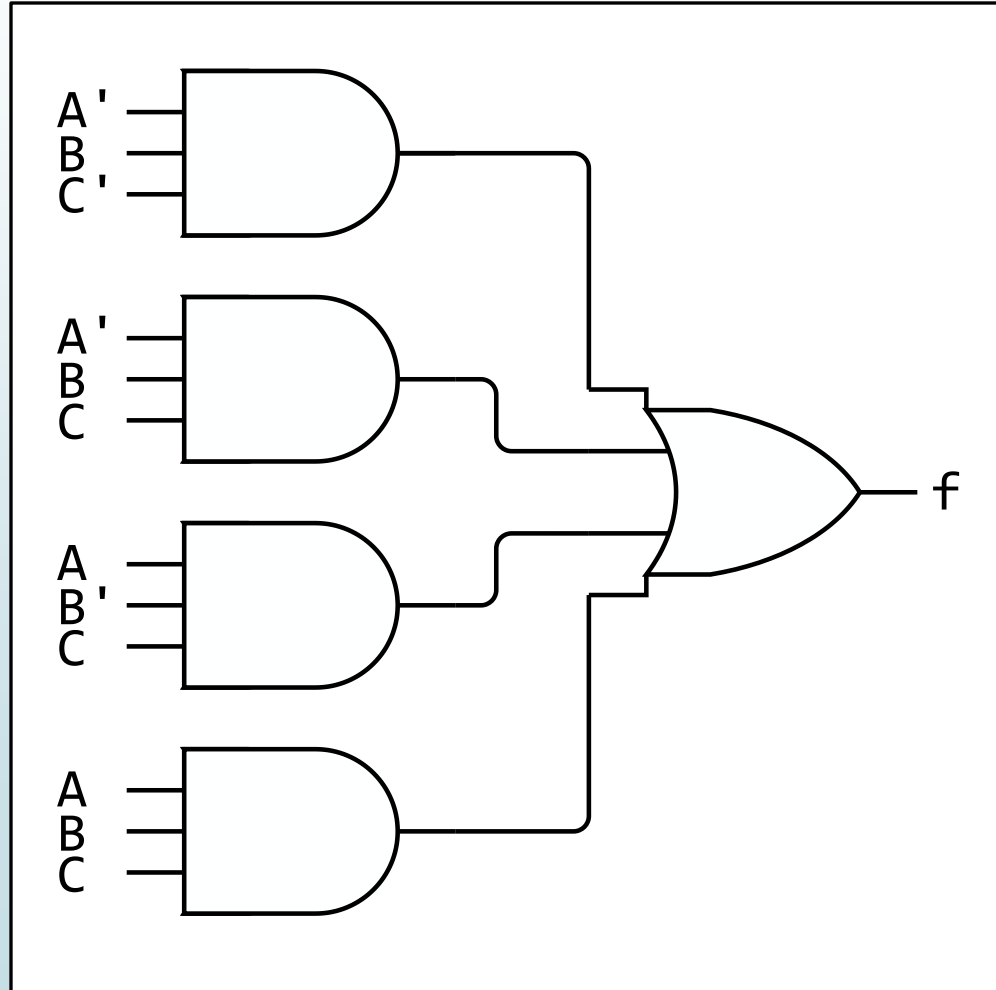
$$f = A'BC' + A'BC + AB'C + ABC$$



Logic Circuit

- Convert this equation into circuit:

$$f = A'BC' + A'BC + AB'C + ABC$$

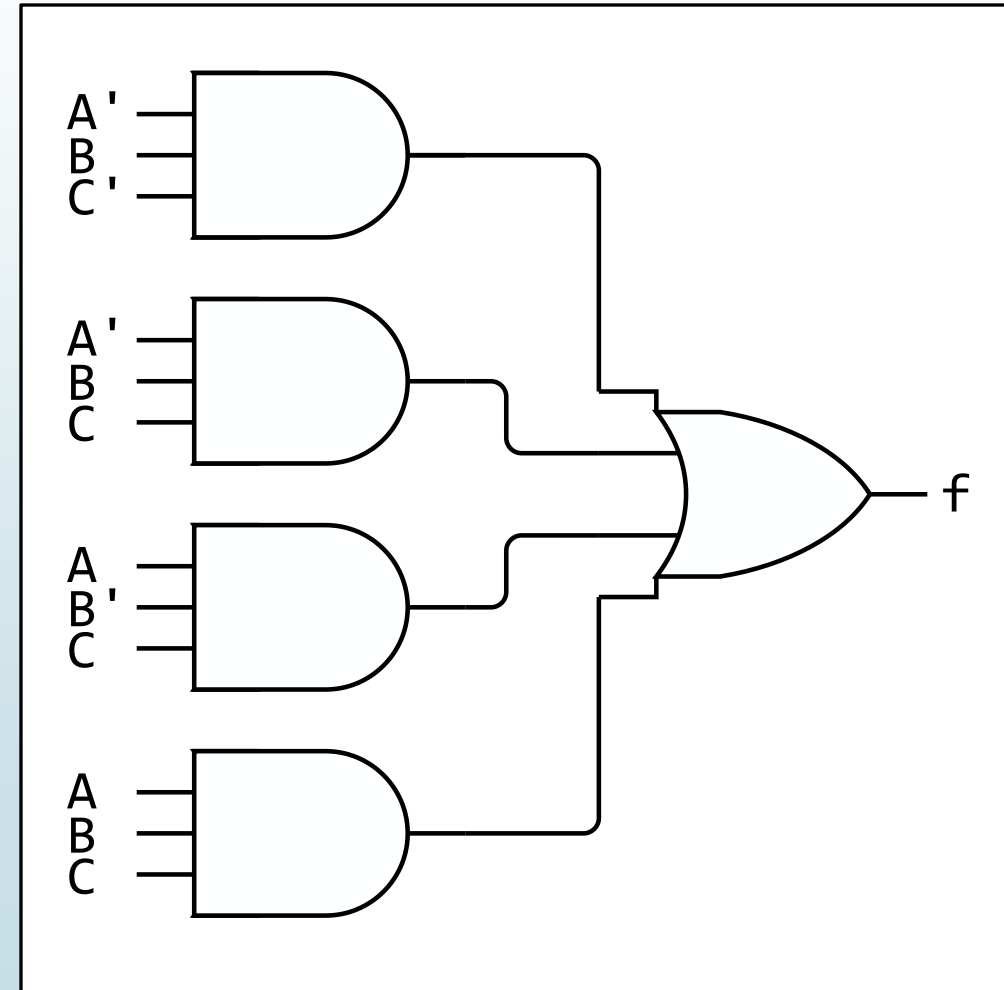


Logic Circuit

- Convert this equation into circuit:

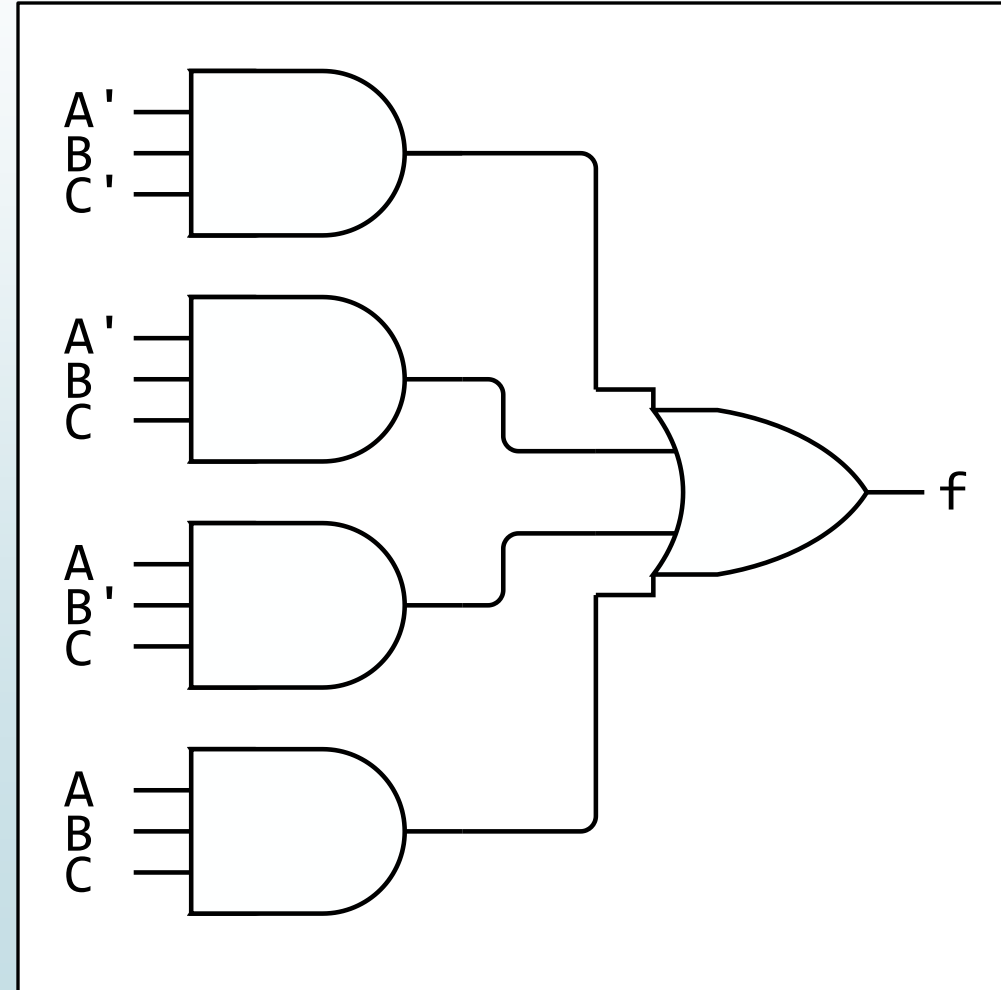
$$f = A'BC' + A'BC + AB'C + ABC$$

- Now, we designed a circuit for the prime number detector.
- However, the design is not perfect. It is too long and complex. It costs many gates.
- We will discuss how to minimize it later.



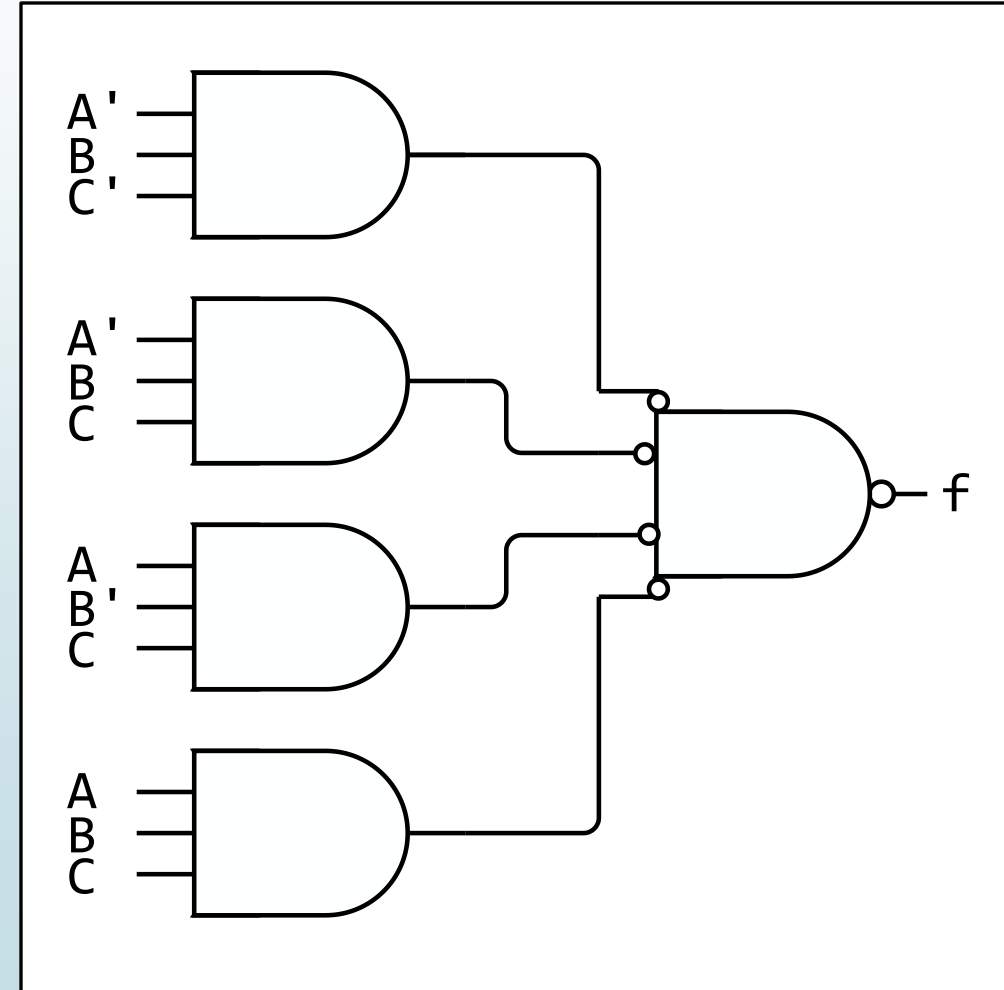
Logic Circuit

- This is an AND-OR circuit.
- The implementation of sum of product equation.



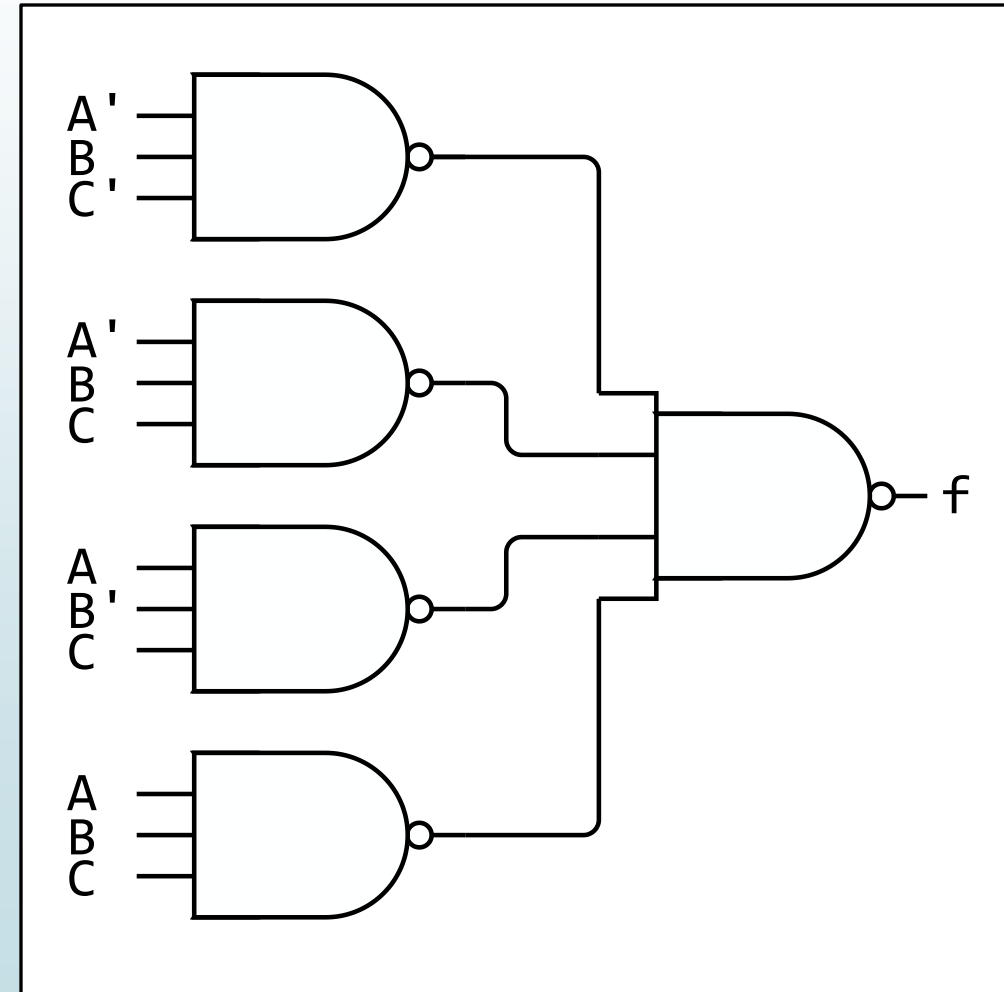
Logic Circuit

- This is an AND-OR circuit.
- The implementation of sum of product equation.
- One cost reduction step is to apply the graphical DeMorgans law on the OR gate.
 - Change the OR to AND.
 - Complement all its inputs and output.



Logic Circuit

- This is a NAND-NAND circuit equivalent to the same function.
- Due to the gate components, the NAND gate is usually cheaper than the AND and OR gates for the same number of inputs.
- Due to the gate components, the NAND gate is usually faster, smaller and consumes less power for the same number of inputs.



Maxterms

- Another way to find the expression is to consider the input cases that makes $f=0$.

A	B	C	Minterms	Maxterms	f
0	0	0	$A'B'C' = m_0$	$A+B+C = M_0$	0
0	0	1	$A'B'C = m_1$	$A+B+C' = M_1$	0
0	1	0	$A'BC' = m_2$	$A+B'+C = M_2$	1
0	1	1	$A'BC = m_3$	$A+B'+C' = M_3$	1
1	0	0	$AB'C' = m_4$	$A'+B+C = M_4$	0
1	0	1	$AB'C = m_5$	$A'+B+C' = M_5$	1
1	1	0	$ABC' = m_6$	$A'+B'+C = M_6$	0
1	1	1	$ABC = m_7$	$A'+B'+C' = M_7$	1

Maxterms

- If any input of them happened, its maxterm is 0, then $f=0$.

$$f = (A+B+C) \cdot (A+B+C') \cdot (A'+B+C) \cdot (A'+B'+C)$$

A	B	C	Minterms	Maxterms	f
0	0	0	$A'B'C' = m_0$	$A+B+C = M_0$	0
0	0	1	$A'B'C = m_1$	$A+B+C' = M_1$	0
0	1	0	$A'BC' = m_2$	$A+B'+C = M_2$	1
0	1	1	$A'BC = m_3$	$A+B'+C' = M_3$	1
1	0	0	$AB'C' = m_4$	$A'+B+C = M_4$	0
1	0	1	$AB'C = m_5$	$A'+B+C' = M_5$	1
1	1	0	$ABC' = m_6$	$A'+B'+C = M_6$	0
1	1	1	$ABC = m_7$	$A'+B'+C' = M_7$	1

Maxterms

- Try all the inputs (prime number) and check the output.

$$f = (A+B+C) \cdot (A+B+C') \cdot (A'+B+C) \cdot (A'+B'+C)$$

A	B	C	Minterms	Maxterms	f
0	0	0	$A'B'C' = m_0$	$A+B+C = M_0$	0
0	0	1	$A'B'C = m_1$	$A+B+C' = M_1$	0
0	1	0	$A'BC' = m_2$	$A+B'+C = M_2$	1
0	1	1	$A'BC = m_3$	$A+B'+C' = M_3$	1
1	0	0	$AB'C' = m_4$	$A'+B+C = M_4$	0
1	0	1	$AB'C = m_5$	$A'+B+C' = M_5$	1
1	1	0	$ABC' = m_6$	$A'+B'+C = M_6$	0
1	1	1	$ABC = m_7$	$A'+B'+C' = M_7$	1

Maxterms

- Try all the inputs (prime number) and check the output.

$$f = (A+B+C) \cdot (A+B+C') \cdot (A'+B+C) \cdot (A'+B'+C)$$

A	B	C	Minterms	Maxterms	f
0	0	0	$A'B'C' = m_0$	$A+B+C = M_0$	0
0	0	1	$A'B'C = m_1$	$A+B+C' = M_1$	0
0	1	0	$A'BC' = m_2$	$A+B'+C = M_2$	1
0	1	1	$A'BC = m_3$	$A+B'+C' = M_3$	1
1	0	0	$AB'C' = m_4$	$A'+B+C = M_4$	0
1	0	1	$AB'C = m_5$	$A'+B+C' = M_5$	1
1	1	0	$ABC' = m_6$	$A'+B'+C = M_6$	0
1	1	1	$ABC = m_7$	$A'+B'+C' = M_7$	1

Maxterms

- Try all the inputs (prime number) and check the output.

$$f = (A+B+C) \cdot (A+B+C') \cdot (A'+B+C) \cdot (A'+B'+C)$$

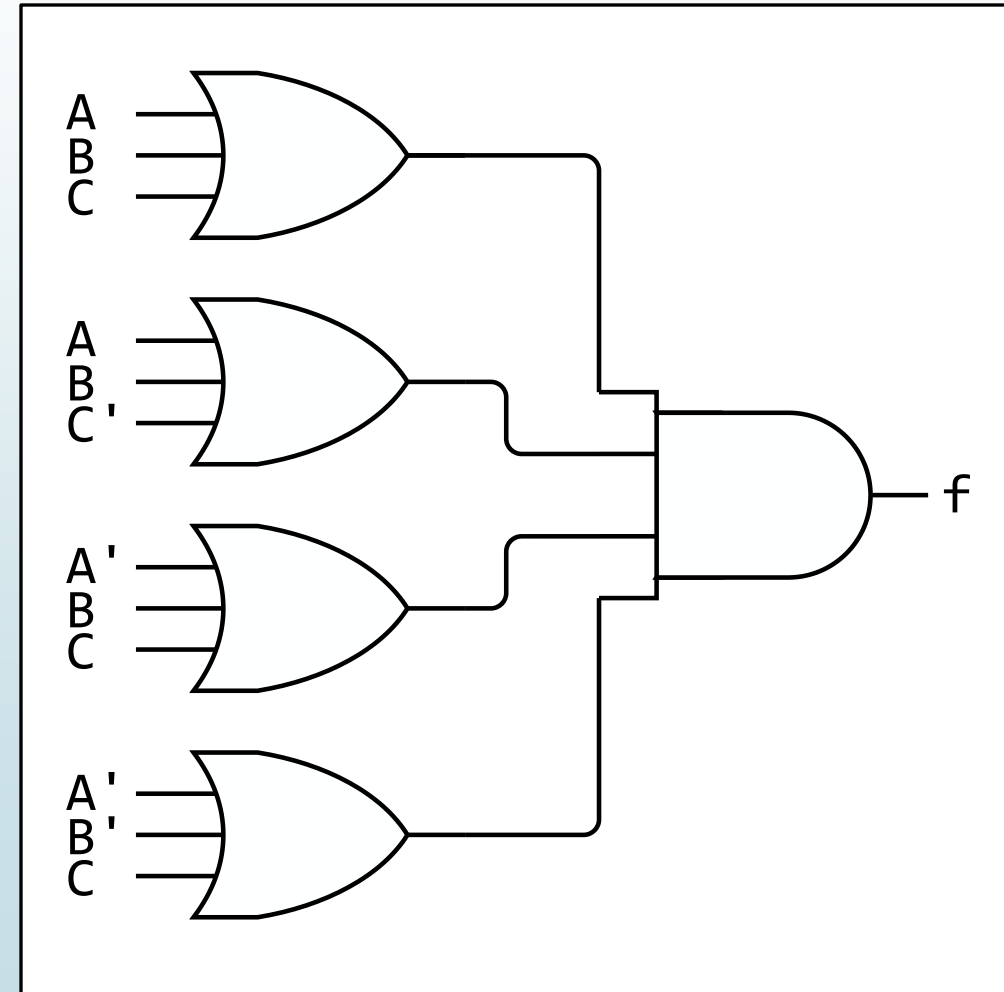
- The expression must be using the AND, because if there is one input of the AND = 0, its output will be 0.
- This is another expression, for the same logic function (detection of prime number).
- It results also in different circuit for the same function.
- This is called the **Product of Sum** form.
- Each of the 4 maxterms is called **sum term**. It is the logical sum (OR) of many variables.

Logic Circuit

- Convert this equation to circuit:

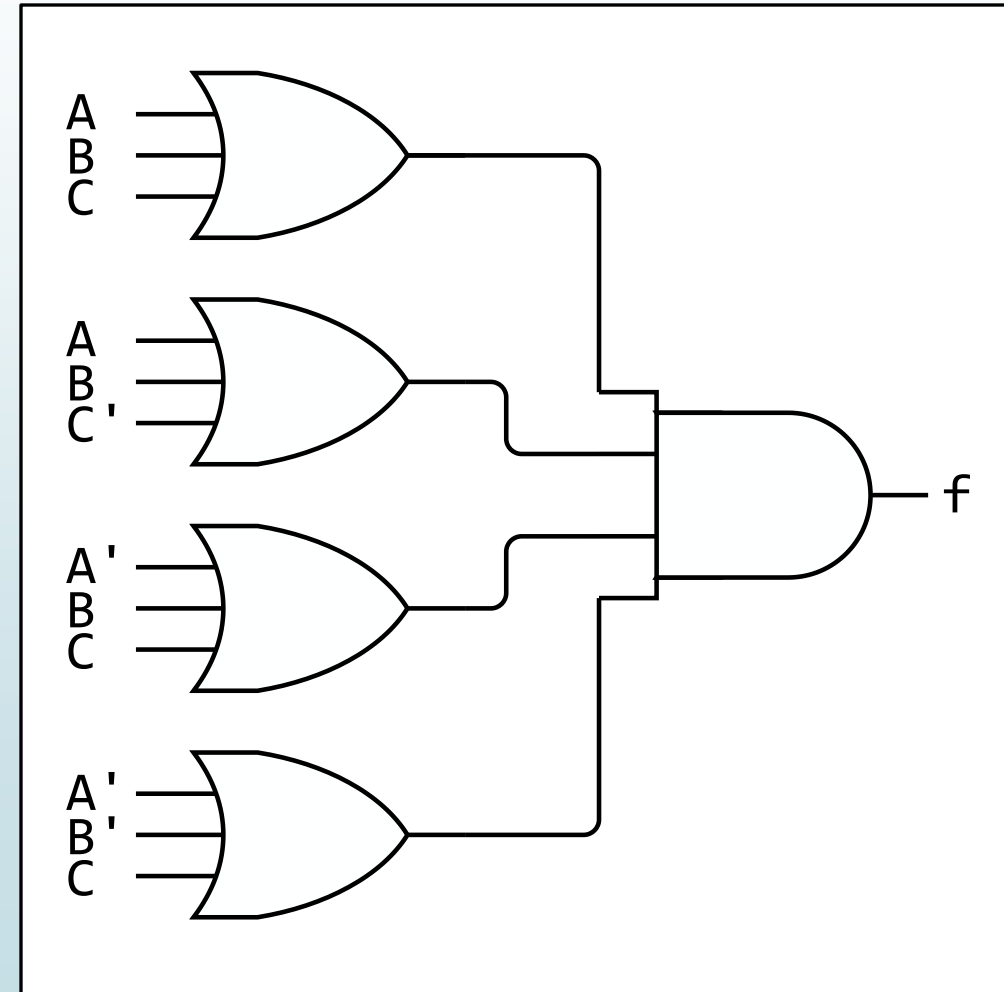
$$f = (A+B+C) \cdot (A+B+C') \cdot (A'+B+C) \cdot (A'+B'+C)$$

- Now, we designed another circuit for the prime number detector.
- However, the design is not perfect. It is too long and complex. It costs many gates.



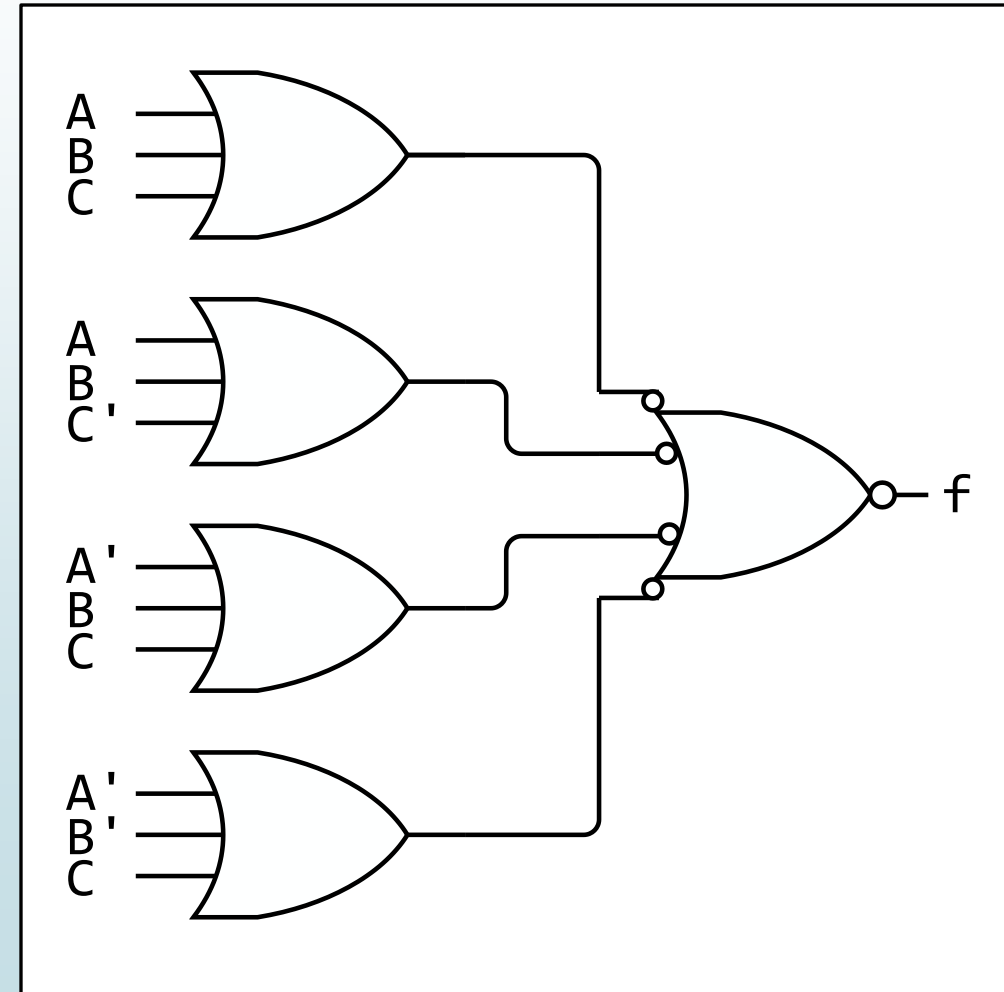
Logic Circuit

► This is an OR-AND circuit.



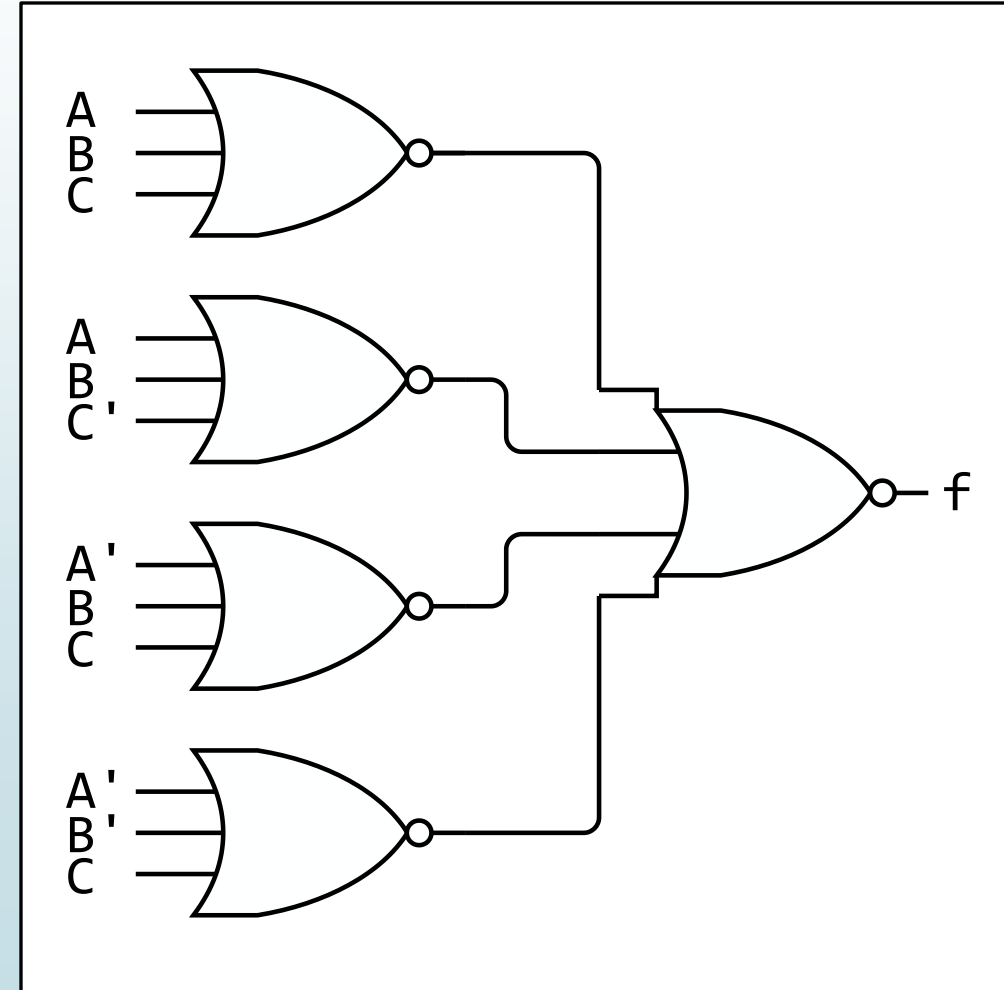
Logic Circuit

- This is an OR-AND circuit.
- One cost reduction step is to apply the graphical DeMorgans law on the OR gate.
 - Change the AND to OR.
 - Complement all its inputs and output.



Logic Circuit

- This is a NOR-NOR circuit equivalent to the same function.
- Due to the gate components, the NOR gate is usually cheaper than the AND and OR gates for the same number of inputs.
- Due to the gate components, the NOR gate is usually faster, smaller and consumes less power for the same number of inputs.



Definitions

- **Product term:** a term consists of the logical product of many variable, either complemented or not. Example:
 $A' B' CD$
- **Sum of Products form:** Any logical sum of product terms.
 - Example: $AB' + BC$
- **Canonical Sum of products of a circuit:** The sum of product form of the circuit function, where output = 1.
 - Example: $f = A'BC' + A'BC + AB'C + ABC$
- **On-set:** the minterms list the turns on the output.
 - Example: $f = \sum_{A,B,C} m(2,3,5,7)$

Definitions

- **Sum term:** a term consists of the logical sum of many variable, either complemented or not. Example: $A' + B' + C + D$
- **Product of Sums form:** Any logical product of sum terms.
 - Example: $(A + B') \cdot (B + C)$
- **Canonical product of sums of a circuit:** The product of sums form of the circuit function, where output = 0.
 - Example: $f = (A + B + C) \cdot (A + B + C') \cdot (A' + B + C) \cdot (A' + B' + C)$
- **Off-set:** the maxterms list the turns off the output.
 - Example: $f = \prod_{A,B,C} M(0,1,4,6)$

Logic Circuit Description

► Many forms to represent a logical circuit:

► Truth table.

► Canonical sum of products: $f = A'BC' + A'BC + AB'C + ABC$

► On-set: $f = \sum_{A,B,C} m(2,3,5,7)$

► Canonical product of sums: $f = (A+B+C) \cdot (A+B+C') \cdot (A'+B+C) \cdot (A'+B'+C)$

► Off-set: $f = \prod_{A,B,C} M(0,1,4,6)$

► Circuit Diagram.

A	B	C	f
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Question ??

- Give the following expression in the standard sum of products form.

$$f(C,B,A) = \sum m(0,3,6)$$

- Answer (Go back to the table):

- $f = m_0 + m_3 + m_6$

$$m_0 = m_{000} \rightarrow C=0, B=0, A=0$$

- $f = C'B'A' + m_3 + m_6$

$$m_3 = ??$$

Question ??

- Give the following expression in the standard sum of products form.

$$f(C,B,A) = \sum m(0,3,6)$$

- Answer (Go back to the table):

- $f = m_0 + m_3 + m_6$

- $f = C'B'A' + m_3 + m_6$

- $f = C'B'A' + C'BA + m_6$

$$m_0 = m_{000} \rightarrow C=0, B=0, A=0$$

$$m_3 = m_{011} \rightarrow C=0, B=1, A=1$$

$$m_6 = ??$$

Question ??

- Give the following expression in the standard sum of products form.

$$f(C,B,A) = \sum m(0,3,6)$$

- Answer (Go back to the table):

- $f = m_0 + m_3 + m_6$

- $f = C'B'A' + m_3 + m_6$

- $f = C'B'A' + C'BA + m_6$

- $f = C'B'A' + C'BA + CBA'$

$$m_0 = m_{000} \rightarrow C=0, B=0, A=0$$

$$m_3 = m_{011} \rightarrow C=0, B=1, A=1$$

$$m_6 = m_{110} \rightarrow C=1, B=1, A=0$$