

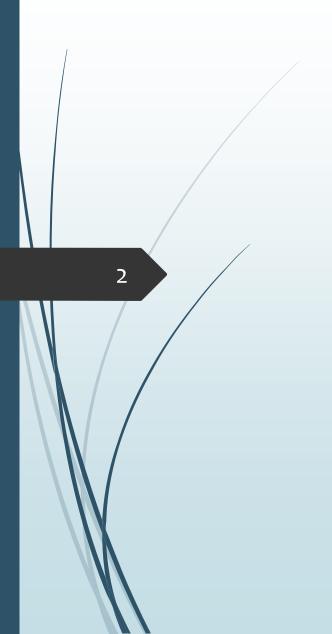




Digital Engineering

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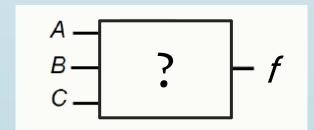
Second Year –Information Technology Program Fall 2025



Lecture 3 Truth Table

Logic Circuit Design

- You will be given a word description for the function required to be implemented.
- **Example:**
 - Design a 3-bit prime number detector circuit.
 - Design a prime number detector circuit for the numbers 0-7.
 - ■Given a 3-bit input N = ABC, design a circuit that will output f=1 in case of N = prime number in binary, and output f=0 otherwise.



- The first step is to construct the circuit truth table.
- It tells what the function will output in each case of input.
- How many inputs ??
- Remember: An n binary bit is used to represent 2ⁿ numbers with the values $0 \rightarrow 2^{n-1}$
- The input range is 0-7. Then, we have 3 inputs (3 binary bit ABC).

| Α | В | С | f |
|---|---|---|---|
| | | | |
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- Regardless of the circuit or its functionality.
- In the input side we need to put all the input combinations.
- How to fill those combinations?

| A | В | С | f |
|---|---|---|---|
| 0 | 0 | 0 | |
| 0 | 0 | 1 | |
| 0 | 1 | 0 | |
| 0 | 1 | 1 | |
| 1 | 0 | 0 | |
| 1 | 0 | 1 | |
| 1 | 1 | 0 | |
| 1 | 1 | 1 | |

- The number of rows = 2^n , where n is the number of inputs.
- ► Here n=3 then the number of $rows = 2^3 = 8$

| A | В | С | f |
|---|---|---|---|
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |

In the right most bit, fill in 0,1,0,1,0,1,... from top to down.

| A | В | C | f |
|---|---|---|---|
| | | 0 | |
| | | 1 | |
| | | 0 | |
| | | 1 | |
| | | 0 | |
| | | 1 | |
| | | 0 | |
| | | 1 | |

- In the right most bit, fill in 0,1,0,1,0,1,... from top to down.
- In the second right most bit, fill in 0,0, 1,1, 0,0, 1,1, ... from top to down.

| Α | В | С | f |
|---|---|---|---|
| | 0 | 0 | |
| | 0 | 1 | |
| | 1 | 0 | |
| | 1 | 1 | |
| | 0 | 0 | |
| | 0 | 1 | |
| | 1 | 0 | |
| | 1 | 1 | |

- In the right most bit, fill in 0,1,0,1,0,1,... from top to down.
- In the second right most bit, fill in 0,0, 1,1, 0,0, 1,1, ... from top to down.
- In the next bit, fill in 0,0,0,0,
 1,1,1,1, ... top to down.
- Repeat till the left most bit.

| A | В | С | f |
|---|---|---|---|
| 0 | 0 | 0 | |
| 0 | 0 | 1 | |
| 0 | 1 | 0 | |
| 0 | 1 | 1 | |
| 1 | 0 | 0 | |
| 1 | 0 | 1 | |
| 1 | 1 | 0 | |
| 1 | 1 | 1 | |

Till now, we constructed the input side only. It is the same in any 3-input circuit.

Now, it is required to construct the output side.

| Α | В | С | f |
|---|---|---|---|
| 0 | 0 | 0 | |
| 0 | 0 | 1 | |
| 0 | 1 | 0 | |
| 0 | 1 | 1 | |
| 1 | 0 | 0 | |
| 1 | 0 | 1 | |
| 1 | 1 | 0 | |
| 1 | 1 | 1 | |

From the word description of the circuit, the output f=1 in case of N = prime number in binary, and the output f=0 otherwise.

| ecimal | |
|--------|--|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| | |

| A | В | C | f |
|---|---|---|---|
| 0 | 0 | 0 | |
| 0 | 0 | 1 | |
| 0 | 1 | 0 | |
| 0 | 1 | 1 | |
| 1 | 0 | 0 | |
| 1 | 0 | 1 | |
| 1 | 1 | 0 | |
| 1 | 1 | 1 | |

From the word description of the circuit, the output f=1 in case of N = prime number in binary, and the output f=0 otherwise.

■ Done with the truth table.

| Decim | |
|-------|--|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| _ | |

| A | В | С | f |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

This is the first step in designing a circuit.

- The next step is to convert it into circuit. How?
- We will learn many ways along the course.

| • | - |
|--------|---|
| cima | |
| CTIIIa | 4 |

0

1

2

3

4

5

6

7

| Α | В | С | f |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

- This is another case.
- Given a function equation. Example: $F = A \cdot \overline{B} + C$
- It is required to find its truth table.

■ How many inputs ??

- This is another case.
- Given a function equation. Example: $F = A \cdot \overline{B} + C$
- It is required to find its truth table.

| Α | В | С | F |
|---|---|---|---|
| | | | |
| | | | |
| | | | |
| | | | |

- \blacksquare How many inputs ?? n=3 (ABC)
- ► How many rows ??

- This is another case.
- Given a function equation. Example: $F = A \cdot \overline{B} + C$
- It is required to find its truth table.

| How many | inputs | ?? n=3 (| (ABC) |
|----------|--------|----------|-------|
|----------|--------|----------|-------|

- How many rows $?? 2^n=8$
- How to fill the input side (ABC columns) ??

| Α | В | С | F |
|---|---|---|---|
| | | | |
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| | | | |

- This is another case.
- Given a function equation. Example: $F = A \cdot \overline{B} + C$
- It is required to find its truth table.

| ■ How mai | ny input | ts ?? n=3 | (ABC) |
|-----------|----------|-----------|-------|
| | | | • |

- \blacksquare How many rows ?? $2^n=8$
- ► How to fill the input side (ABC columns) ??

| Α | В | C | F |
|---|---|---|---|
| | | 0 | |
| | | 1 | |
| | | 0 | |
| | | 1 | |
| | | 0 | |
| | | 1 | |
| | | 0 | |
| | | 1 | |

- This is another case.
- Given a function equation. Example: $F = A \cdot \overline{B} + C$
- It is required to find its truth table.

| ■ How many | inputs | ?? n=3 | (ABC) |
|------------|--------|--------|-------|
|------------|--------|--------|-------|

- \blacksquare How many rows ?? $2^n=8$
- ► How to fill the input side (ABC columns) ??

| Α | В | С | F |
|---|---|---|---|
| | 0 | 0 | |
| | 0 | 1 | |
| | 1 | 0 | |
| | 1 | 1 | |
| | 0 | 0 | |
| | 0 | 1 | |
| | 1 | 0 | |
| | 1 | 1 | |

- This is another case.
- Given a function equation. Example: $F = A \cdot \overline{B} + C$
- It is required to find its truth table.

- \blacksquare How many inputs ?? n=3 (ABC)
- How many rows $?? 2^n=8$
- ► How to fill the input side (ABC columns)??

| Α | В | С | F |
|---|---|---|---|
| 0 | 0 | 0 | |
| 0 | 0 | 1 | |
| 0 | 1 | 0 | |
| 0 | 1 | 1 | |
| 1 | 0 | 0 | |
| 1 | 0 | 1 | |
| 1 | 1 | 0 | |
| 1 | 1 | 1 | |

- This is another case.
- Given a function equation. Example: $F = A \cdot \overline{B} + C$

- How do you think we could fill in the output side??
- Previously we had a word description. Now how??

| Α | В | С | F |
|---|---|---|---|
| 0 | 0 | 0 | |
| 0 | 0 | 1 | ; |
| 0 | 1 | 0 | ? |
| 0 | 1 | 1 | ? |
| 1 | 0 | 0 | ? |
| 1 | 0 | 1 | ? |
| 1 | 1 | 0 | ? |
| 1 | 1 | 1 | ? |

- This is another case.
- Given a function equation. Example: $F = A \cdot \overline{B} + C$
- It is required to find its truth table.

■ Substitute in the equation.

| Α | В | С | F |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | |
| 0 | 1 | 0 | |
| 0 | 1 | 1 | |
| 1 | 0 | 0 | |
| 1 | 0 | 1 | |
| 1 | 1 | 0 | |
| 1 | 1 | 1 | |

- This is another case.
- Given a function equation. Example: $F = A \cdot \overline{B} + C$
- It is required to find its truth table.

■ Substitute in the equation.

| A | В | С | F |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | |
| 0 | 1 | 1 | |
| 1 | 0 | 0 | |
| 1 | 0 | 1 | |
| 1 | 1 | 0 | |
| 1 | 1 | 1 | |

- This is another case.
- Given a function equation. Example: $F = A \cdot \overline{B} + C$
- It is required to find its truth table.

- Substitute in the equation.
- **■** Done.

| Α | В | С | F |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

- Remember: To get the dual of a function: Replace operators:
 + ↔ · and replace constants
 0 ↔ 1: (Variables are not changed).
 - ightharpoonup Example: $F = A \cdot \overline{B} + C$
 - Then: $F^D = (A + \bar{B}) \cdot C$
- \blacksquare Given the truth table of F.
- $lue{}$ How do you think could we get the truth table of F^D ??

| Α | В | С | F |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

- Remember: To get the dual of a function: Replace operators:
 + ↔ · and replace constants
 0 ↔ 1: (Variables are not changed).
- From this rule, replace all constants $0 \leftrightarrow 1$
- This is the truth table but with reversed rows.

| A | В | С | F |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 0 | 0 | 0 |

- Remember: To get the dual of a function: Replace operators:
 + ↔ · and replace constants
 0 ↔ 1: (Variables are not changed).
- From this rule, replace all constants $0 \leftrightarrow 1$
- This is the truth table but with reversed rows.
- Rearrange the rows. Reverse them up down.

| Α | В | С | F |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

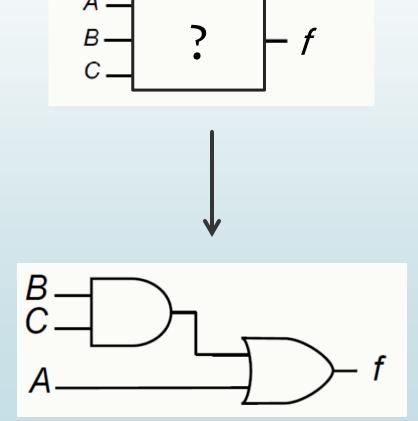
Check it by substituting in the equation $F^D = (A + \overline{B}) \cdot C$

Done.

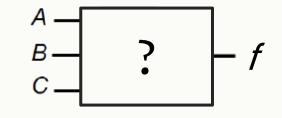
| A | В | С | F |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

- This is another case.
- This is reverse engineering.
- Given an unknown chip and it is required to find its truth table, equation and circuit diagram.

■ First find the truth table and then use any of the upcoming methods to find the equation and circuit.

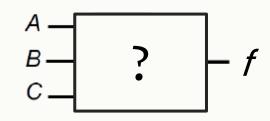


First construct the input side using the same previous steps.



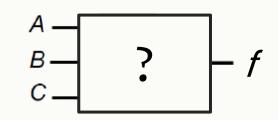
■ How many inputs ?? n=?

- First construct the input side using the same previous steps.
- \blacksquare How many inputs ?? n=3 (ABC)
- How many rows ??



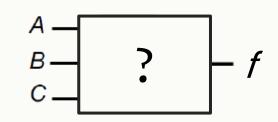
| Α | В | С | F |
|---|---|---|---|
| | | | |
| | | | |
| | | | |

- First construct the input side using the same previous steps.
- \blacksquare How many inputs ?? n=3 (ABC)
- \rightarrow How many rows ?? $2^n=8$
- How to fill the input side (ABC columns) ??



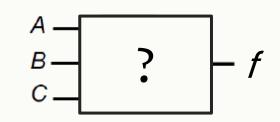
| A | В | С | F |
|---|---|---|---|
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- First construct the input side using the same previous steps.
- \blacksquare How many inputs ?? n=3 (ABC)
- \rightarrow How many rows ?? $2^n=8$
- How to fill the input side (ABC columns)?
- ► How do you think could we fill in the output side ??



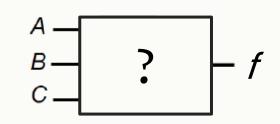
| Α | В | C | F |
|---|---|---|---|
| 0 | 0 | 0 | ; |
| 0 | 0 | 1 | ; |
| 0 | 1 | 0 | ; |
| 0 | 1 | 1 | ; |
| 1 | 0 | 0 | ; |
| 1 | 0 | 1 | ; |
| 1 | 1 | 0 | ; |
| 1 | 1 | 1 | ; |

- First construct the input side using the same previous steps.
- \blacksquare How many inputs ?? n=3 (ABC)
- \rightarrow How many rows ?? $2^n=8$
- How to fill the input side (ABC columns)?
- How do you think could we fill in the output side ?
- Do it empirically: Apply each input and measure the output.



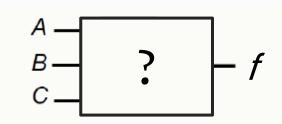
| Α | В | C | F |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | |
| 0 | 1 | 0 | |
| 0 | 1 | 1 | |
| 1 | 0 | 0 | |
| 1 | 0 | 1 | |
| 1 | 1 | 0 | |
| 1 | 1 | 1 | |

- First construct the input side using the same previous steps.
- \blacksquare How many inputs ?? n=3 (ABC)
- \rightarrow How many rows ?? $2^n=8$
- How to fill the input side (ABC columns)?
- How do you think could we fill in the output side ?
- Do it empirically: Apply each input and measure the output.



| A | В | С | F |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | |
| 0 | 1 | 1 | |
| 1 | 0 | 0 | |
| 1 | 0 | 1 | |
| 1 | 1 | 0 | |
| 1 | 1 | 1 | |

- First construct the input side using the same previous steps.
- \blacksquare How many inputs ?? n=3 (ABC)
- \rightarrow How many rows ?? $2^n=8$
- How to fill the input side (ABC columns)?
- How do you think could we fill in the output side?
- Do it empirically: Apply each input and measure the output. Done.



| Α | В | С | F |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |